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Jonas A. Zukas

Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

November 1969

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BRL R 1457

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110

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Terminal Ballistics Laboratory

RDT&E Project No. 1B062104A088

ABERDEEN PROVING GROUND, MARYLAND

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ABSTRACT

A theory for the analysis of stresses in laminated circular cylindrical shells subjected to arbitrary axisymmetric mechanical and thermal loadings has been developed. This theory, specifically for use with pyrolytic graphite type materials, differs from the classical thin shell theory in that it includes the effects of transverse shear deformation and transverse isotropy, as well as thermal expansion through the shell thickness.

Solutions in several forms are developed for the governing equations. The form taken by the solution function is governed by geometric considerations. A range in which the various solution forms occur was determined numerically.

As a sample problem, the slow cooling of pyrolytic graphite deposited onto a commercial graphite mandrel was considered. Investigation of normal and shear stress behavior at the pyrolytic graphite - mandrel interface showed that these stresses decrease in magnitude with increasing E/G_c ratio and increasing deposit to mandrel thickness (h_a/h_b) ratio. This implies that a thin mandrel and a material weak in shear are desirable to minimize the possibilities of flaking and delamination of the pyrolytic graphite.

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TABLE OF CONTENTS

NOTATION

a, b	subscripts indicating upper or lower lamina
a_{ij}, b_{ij}	constants defined by equations (C.2) and (C.3)
A,B	constants defined by (C.8)
c	preferred direction in a transversely isotropic material
c_i	roots of equation (22) defined by (C.11) and (C.13)
C_i	$\frac{E_i h_i}{1 - \nu_i^2}$ (i = a,b)
D_i	$\frac{E_i h_i^3}{12(1 - \nu_i^2)}$
D	d/dx
e	natural base
E, E_c	Young's Modulus in the plane of isotropy and "c" direction respectively
G_c	shear modulus relating stress and strain across the plane of isotropy ($=G_{xz} = G_{\theta z} = G_{zx} = G_{z\theta}$)
g_i	constants defined by equation (C.4)
h_a, h_b	individual lamina thicknesses
h	$h_a + h_b$
i, j	subscripts
k_i	constants defined by equation (C.6)
m, n	constants defined by equation (C.7)
$\bar{m}$	constant defined by (11a)
M_x,M_θ	stress couples

M_{Tx}, $M_{T\theta}$	thermal couples
$\overline{M}$	defined by equation (41)
N_x, N_θ	stress resultants
N_{Tx}, $N_{T\theta}$	thermal resultants defined by equation (41)
p_{1i}	$\sigma_z\ (h_i/2)\quad (i = a,b)$
p_{2i}	$\sigma_z\ (-h_i/2)\quad (i = a,b)$
$p(x)$	$p_{1a} - p_{2b}$
p_j	joint normal stress $(=\sigma_z\ (-\frac{h_a}{2}) = \sigma_z\ (+\frac{h_b}{2})$
Q_i	shear resultant
$\overline{Q}$	defined by equation (41)
R	Radius to shell reference surface
T	temperature measured from the stress-free temperature of the material
u_x, u_z	deflections in the "x" and "z" directions respectively
uo_i	axial deflection of lamina middle surface
u_i^*	virtual displacement of shell middle surface
w_i	radial displacement of lamina middle surface
$\overline{w}_i$	$\int_0^z (\alpha_c T)_i\ dz\quad (i = a,b)$
w_i^*	virtual radial displacement of lamina middle surface.
x	axial coordinate for cylindrical shell
z	cylindrical coordinate
α,α_c	thermal expansion coefficients in the "x" and "z" directions respectively

β_i	rotation of the normal to the undeformed lamina middle surface due to deformation
Γ	defined by equation (C.10)
$\lambda_1, \lambda_2, \lambda_3$	defined by equation (C.9)
β_i^*	virtual rotation
Δ_i	defined by equation (21)
ε_{ij}	strain component
ν	Poisson's ratio in the $x - \theta$ plane ($\nu_{x\theta} = \nu_{\theta x}$)
ν_c	Poisson's ratio ($\nu_{xz} = \nu_{\theta z}$)
ν_{ij}	Poisson's ratio defined as the negative of the ratio of the strain in the j-direction to the strain in the i-direction due to a stress in the i-direction
σ_{ij}	stress component
τ_j	joint shear stress ($=\sigma_{zx}(-h_a/2) = \sigma_{zx}(+h_b/2)$)
θ	cylindrical coordinate in the circumferential direction
θ_1, θ_2	defined by equation (C.10)
$(\ )'$	$\frac{d}{dx}(\ )$

I. INTRODUCTION

The ever- expanding missile and space technology continually demands materials capable of maintaining structural integrity at very high temperatures. Of late, attention has been focused on refractory materials, their anisotropy in physical and mechanical properties making them ideally suited for a wide range of insulation and/or structural applications.

Of the many refractory materials possible, pyrolytic graphite (PG) has probably received the most attention of late although it was known to Edison (1)* in 1883 who described methods for its manufacture, the technique involving formation of carbon deposits onto substrates heated in carbon-containing gases. For structural use, pyrolytic graphite is generally deposited at temperature from 3500°F to 4000°F in a stream of hydrocarbon gas, such as methane, onto a substrate of commercial graphite maintained at temperatures of 1500°F to 5000°F. The rate at which the material is produced depends on a number of factors which include the temperature, the reaction pressure, the hydrocarbon flow rate and

*Numbers in parenthesis indicate corresponding references in the bibliography

the surface to volume ratio of the substrate surface (2), (3), (4), (5). X-ray analysis of the resultant deposit shows a well-crystallized structure having much in common with the single graphite crystal (6). Growth is always normal to the substrate surface and after a thickness of 0.1" - 0.5" is reached, the deposition process is stopped and the deposit allowed to cool for several weeks.

The result of such a formation process is a material highly anisotropic in physical properties. The PG has one plane of isotropy parallel to the mandrel surface (x,θ direction -- see figure II-1 for geometry) and a single preferred direction (z), a state commonly referred to as transverse isotropy. With thermal conductivity between 100 and 1000 times greater in the (x,θ) direction than in the (z) direction, the material acts as an excellent conductor along its surface but also as a good insulator in the thickness (z) direction. The coefficient of thermal expansion in the (z) direction is from 10 to 30 times greater than that in the plane of isotropy (x,θ) so that thermal expansion through-the-thickness must be considered in many analyses of the material's thermal behavior.

Other curious effects due to the anisotropy are manifest in the Poisson's ratio which is negative in the plane of isotropy ($\nu_{x\theta} = \nu_{\theta x} = -0.21$) but large and positive in the preferred direction

($\nu_{xz} = \nu_{\theta z} = +0.9$). Furthermore, the ratio of the elastic modulus in the isotropic plant to the shear modulus in the transverse plane (E_x/G_{xz} or $E_\theta/G_{\theta z}$) may range from 20 to 50, compared to an E/G ratio of 2.5 for an isotropic material with $\nu = 0.25$. Therefore, in an analysis of a structure composed of such material, transverse shear deformation even for thin cross-sections must be considered.

Thermal and mechanical properties can be found readily (2), (5), (7), (8), (9), (10), (11), (12). Typical properties, given in Table I-1, are taken from (10), which are reasonably close to those given in other references, discrepancies most probably being due to variations in the deposition process.

Among the earliest analyses of structures of pyrolytic materials were those of Garber (13) and Levy (14) who treated thermal stresses in cylindrical and spherical shells and also the residual stresses caused by the general anisotropy of pyrolytic graphite, but neglected transverse shear deformation and did not account for the high thermal expansion coefficient in the (z) direction. McDonough (15) has considered thermal stresses in shells of revolution of pyrolytic graphite type materials subjected to axially symmetric loads, including transverse shear deformation and thermal expansion through the thickness. He was able to show that neglect of transverse shear deformation would lead to an over-estimate of the stiffness coefficient

TABLE I-1 PHYSICAL PROPERTIES OF PYROLYTIC GRAPHITE

MECHANICAL PROPERTIES

1. Young's Modulus (PSI)

TEMP	(x,θ) Direction	(z) Direction
70°F	5.4×10^6	$1.5\ \times10^6$
1000°F	4.3×10^6	1.29×10^6
2000°F	3.5×10^6	1.05×10^6
3000°F	2.7×10^6	0.81×10^6

2. Poisson's Ratio

70°F	$\nu_{x\theta} = 0.21$	$\nu_{xz} = 0.90$

THERMAL PROPERTIES

1. Thermal Expansion in/in - °F

70°F	0.0	13.1×10^{-6}
1000°F	0.6×10^{-6}	13.1×10^{-6}
2000°F	1.2×10^{-6}	13.1×10^{-6}
3000°F	1.7×10^{-6}	13.1×10^{-6}

2. Conductivity, BTU/hr-ft-°F

70°F	290.0	1.25
1000°F	165.0	0.82
2000°F	100.0	0.60
3000°F	60.0	0.60

for properties representative of PG while excluding thermal expansion in the (z) direction leads not only to erroneous stress predictions but that even the sign of the stress (tension or compression) may be wrong. Kliger (16), (17) has extended McDonough's work for the case of conical shells in that he derives equations for non-axially symmetric mechanical and thermal loadings. Raju (18) studied the case of shallow shells of pyrolytic graphite type materials subjected to a variety of axially symmetric and non-axially symmetric loads. Daugherty (19), (20) treated the case of non-circular cylindrical shells of pyrolytic materials.

The preceding deal with single-layer shells. Anisotropic laminated shells of revolution with elastic properties symmetric about the middle surface of the composite shell are extensively treated by Ambartsumian (21), whereas Dong (22), (23), et al (24) treat layered shells wherein the structure is assumed to be composed of an arbitrary numbers of bonded layers each of different constant thickness, different orientation of elastic axes and different anisotropic elastic properties. Since Dong does not assume elastic symmetry about the middle surface, flexural and extensional deformations are coupled and solution techniques for homogeneous shells do not carry over directly for anisotropic elastic shells. Hence, alternate methods of solution are developed.

Radkowski et al (25) considered laminated isotropic shells of revolution with variable thickness using E. Reissner's for-

formulation (26). Radkowski extended this work to include variable laminated orthotropic material properties (27). Both formulations were restricted to axisymmetric loads. In Radkowski's works and that of Sepetoski (28), the governing equations were cast in finite difference form and solved with the aid of a digital computer. The introduction to Dong's paper (23) makes interesting reading regarding the hazards of this perfectly valid technique.

Other treatments of laminated cylinders have been by Jones and Whittier (29), Tsai and Azzi (30), Paul (31), Au (32), Keeffe and Windholz (33). These, and most other references cited herein are characterized by neglect of transverse shear deformation. A recent work of great theoretical elegance, even though it neglects transverse shear deformation, is that of Zudans (34) which presents a theory for arbitrarily loaded (mechanically & thermally) shells of revolution with internal masses and ring stiffeners, derived under the Kirchoff Hypothesis and consistent with balance of energy as well as linear and angular momentum and invariance under transformation of middle surface coordinate systems and rigid body displacements. The elegance, unfortunately, does not carry over to the computational techniques (35).

Laminated isotropic plates have been considered by Vinson (36) who treated thermal stresses in circular plates, neglecting transverse shear deformation. Summers (37) treats thick and thin isotropic and orthotropic laminated plates including transverse shear deformation.

Mehta (34) considers orthotropic and isotropic laminated as well as single-layered rectangular plates of pyrolytic graphite type materials under static mechanical and thermal loads. Wu (39), (40), and (41) treats the lateral vibrations of both small and large amplitude for rectangular plates of pyrolytic and graphite materials.

Except for those references dealing with pyrolytic graphite type materials, all the works reviewed which analyze multi-layered structures are characterized by their neglect of either transverse shear deformation or thermal expansion through-the-thickness or both.

Prompted by the absence of a definitive treatment of laminated shells applicable to pyrolytic graphite type materials, this thesis was undertaken. It is an extension of McDonough's work in that layered cylindrical shells, including the effects of transverse shear deformation and thermal expansion through-the-thickness, subjected to arbitrary axi-symmetric loading are considered.

II. DERIVATION OF GOVERNING EQUATIONS

The coordinate system used is shown in Figure (II-1). The three-dimensional equations of thermoelasticity (uncoupled) for the case of axial symmetry are given by:

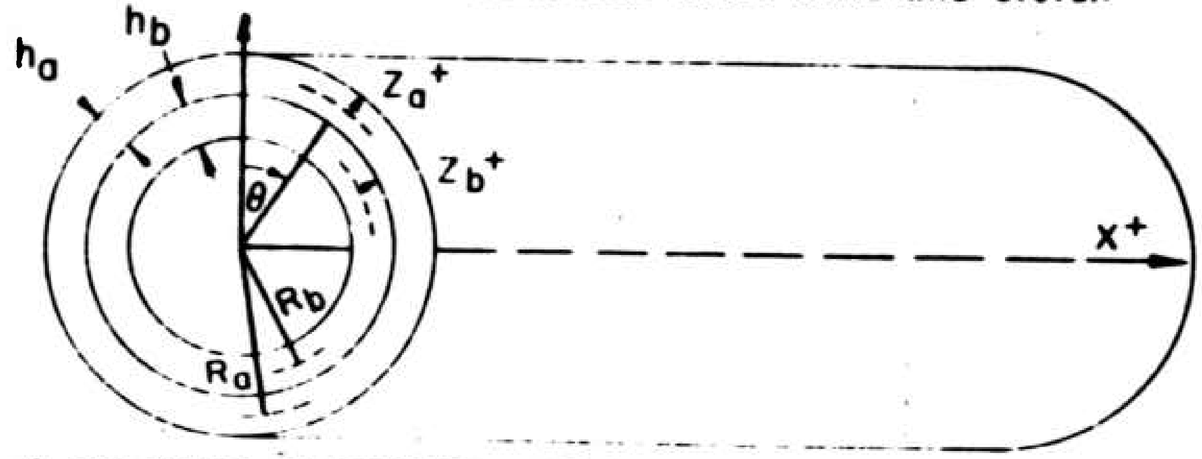
Stress-Strain Relations (transversely isotropic material)

$$\begin{aligned}
\varepsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_\theta - \nu_c\sigma_z) + \alpha T \\
\varepsilon_\theta &= \frac{1}{E}(\sigma_\theta - \nu\sigma_x - \nu_c\sigma_z) + \alpha T \\
\varepsilon_z &= \frac{\sigma_z}{E_c} - \frac{\nu_c}{E}(\sigma_x + \sigma_\theta) + \alpha_c T \\
\varepsilon_{xz} &= \frac{\sigma_{xz}}{2G_c}
\end{aligned} \tag{1}$$

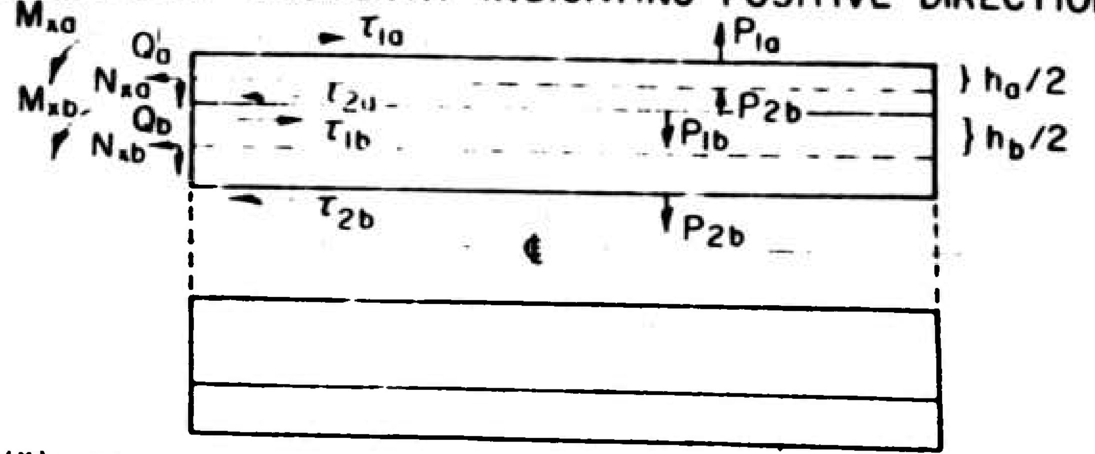
where $\varepsilon_{x\theta} = \varepsilon_{\theta z} = \sigma_{x\theta} = \sigma_{\theta z} = 0$ by symmetry

ε_{ij} and σ_{ij} are the physical components of the strain and stress tensors respectively, and for brevity $\sigma_{ij} = \sigma_i$ when $i = j$. E, E_c, ν, ν_c, G_c are five independent elastic constants where again for brevity $\nu = \nu_{x\theta} = \nu_{\theta x}$ and $\nu_c = \nu_{xz} = \nu_{\theta z}$. α, α_c are the coefficients of thermal expansion in the x and z directions of the materials. T is the temperature measured from the stress-free temperature of the material in units consistent with the α's.

FIGURE II-1: LAYERED CYLINDRICAL SHELL COORDINATE SYSTEM



1A. CYLINDER GEOMETRY INDICATING POSITIVE DIRECTIONS



$P^{(x)} = P_{1a} - P_{2b}$; $\tau_j = \tau_{2a} = \tau_{1b}$; $P_j = P_{2a} = P_{1b}$

1B. FORCES ON LAYERED CYLINDER

The preferred direction for the material is everywhere coincident with the z coordinate. This restriction is carried throughout this work.

Equilibrium Equations

The equilibrium equations when applied to the present problem become for each lamina:

$$R(1 + \frac{z}{R}) \frac{\partial \sigma_x}{\partial x} + R(1 + \frac{z}{R}) \frac{\partial \sigma_{xz}}{\partial z} + \sigma_{xz} = 0$$

$$R(1 + \frac{z}{R}) \frac{\partial \sigma_z}{\partial z} + R(1 + \frac{z}{R}) \frac{\partial \sigma_{xz}}{\partial x} + \sigma_z - \sigma_\theta = 0 \qquad (2)$$

The third equation is identically zero from symmetry considerations.

Strain-Deformation Equations

The strain-deformation relations can be written correspondly as:

$$\varepsilon_x = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_\theta = \frac{1}{R(1+\frac{z}{R})} u_z$$

$$\varepsilon_z = \frac{\partial u_z}{\partial z} \qquad (3)$$

$$\varepsilon_{xz} = \frac{1}{2} \{\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\}$$

$u_\theta \equiv 0$ by symmetry

The displacements are positive in the direction of the positive corresponding coordinate (See Figure II-1).

Assumptions:

1. The thickness of the shell is small compared with other dimensions, hence Love's First Approximation is applicable:

$$\text{i.e.} \quad \frac{h}{R_{min}} << 1 \tag{4}$$

2. The displacements are small compared to the thickness of the shell and the angles of rotation are small compared to unity.

3. The transverse normal stress is small compared with other normal stress components and is neglected in the stress-strain equations.

4. A linear element normal to the undeformed middle surface undergoes translation and rotation and remains straight, implying deformations of the form

$$u_x = u_o(x) + z\beta(x) \tag{5}$$

5. Transverse normal strain due to thermal expansion will be included; that due to mechanical (or isothermal) loads will be neglected.

This implies a deformation of the form

$$u_z(x,z) = w(x) + \bar{w}(x,z) \tag{6}$$

where

$$\bar{w}(x,z) = \int_o^z \alpha_c T dz$$

6. Material properties are constant for each lamina.

With these assumptions, the equation (1) becomes:

$$
\begin{aligned}
\varepsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_\theta) + \alpha T \\
\varepsilon_\theta &= \frac{1}{E}(\sigma_\theta - \nu\sigma_x) + \alpha T, \\
\varepsilon_z &= \alpha_c T \\
\varepsilon_{xz} &= \frac{\sigma_{xz}}{2G_c}
\end{aligned}
\tag{7}
$$

Making use of equations (5) and (6) and neglecting z/R in comparison to unity, the equations (3) become:

$$
\begin{aligned}
\varepsilon_x &= u_0' + z\beta' \\
\varepsilon_\theta &= \frac{1}{R}(w + \overline{w}) \\
\varepsilon_z &= \frac{\partial \overline{w}}{\partial z} \\
\varepsilon_{xz} &= 1/2(\beta + w')
\end{aligned}
\tag{8}
$$

where $(\ )' = D(\ ) = \frac{d}{dx}(\ )$

Integrated Equations

The stress resultants and couples are defined as follows:

$$N_{x_i} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \sigma_x dz \qquad N_{Tx_i} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} E_i \alpha_i T dz$$

$$N_{\theta_i} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \sigma_\theta dz \qquad N_{T\theta_i} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} E_i \alpha_i T dz$$

$$Q_i = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \sigma_{xz} dz \tag{9}$$

$$M_{x_i} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} z\sigma_x dz \qquad M_{Tx_i} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} zE_i \alpha_i T dz$$

$$M_{\theta_i} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} z\sigma_\theta \, dz \qquad M_{T\theta_i} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} zE_i \alpha_i T dz$$

$i = a, b$

The equilibrium equations in terms of stress resultants and couples are readily obtained. Integrating (2) directly yields

$$
\begin{aligned}
&N'_{x_i} + \tau_{1i} - \tau_{2i} = 0 \\
&M'_{x_i} - Q_i + \frac{h_i}{2}\tau_{1i} + \frac{h_i}{2}\tau_{2i} = 0 \\
&Q'_i - \frac{N_{\theta i}}{R_i} + P_{1i} - P_{2i} = 0
\end{aligned}
\qquad (10)
$$

$i = a,b$

Solving equations (7) for normal stresses, first integrating them from $-h_i/2$ to $+ h_i/2$; then multiplying through the equations by z and integrating once again between the same limits, making use of the definitions of the stress resultants and couples (9) and simplifying, the following stress-strain relations result:

$$N_{x_i} = \frac{E_i\, h_i}{(1-\nu_i^2)}\left(u'_{oi} + \frac{\nu_i w_i}{R_i}\right) - \frac{N_{Tx_i}}{1-\nu_i} + \frac{E_i\, \nu_i}{R_i(1-\nu_i^2)} \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \bar{w}_i\, dz$$

$$N_{\theta_i} = \frac{E_i\, h_i}{(1-\nu_i^2)}\left(\nu_i u'_{oi} + \frac{w_i}{R_i}\right) - \frac{N_{T\theta_i}}{1-\nu_i} \pm \frac{E_i}{R_i(1-\nu_i^2)} \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \bar{w}_i\, dz$$

$$M_{x_i} = \frac{E_i h_i^{\,3}}{12(1-\nu_i^2)}\, \beta'_i - \frac{M_{Tx_i}}{1-\nu_i} + \frac{E_i \nu_i}{R_i(1-\nu_i^2)} \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} z\bar{w}_i\, dz \qquad (11)$$

$$M_{\theta_i} = \frac{E_i h_i^{\,3}}{12(1-\nu_i^2)}\, \beta'_i - \frac{M_{T\theta_i}}{1-\nu_i} + \frac{E_i}{R_i(1-\nu_i^2)} \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} z\bar{w}_i\, dz$$

$i = a, b$

An integrated shear stress strain equation is required in addition to (11). The necessary expression is obtained using weighted integration, the procedure being analogous to that in reference 15, and for convenience given in Appendix D.

$$Q_i = \frac{\overline{m}_i}{6} + \frac{5}{6} h_i G_c^i (\beta_i + w_i') + \Delta_i$$

$$\text{where } \overline{m}_i = \frac{h_i}{2}(\tau_{1i} + \tau_{2i}) \quad . \tag{11A}$$

Solving for Q_a and Q_b from the second of the integrated equilibrium equations (10) and substituting into the third, making use of M_{x_i} and N_{θ_i} from (11) and the definitions

$$C_i = \frac{E_i h_i}{(1-\nu_i^2)} \quad ; \quad D_i = \frac{E_i h_i^3}{12(1-\nu_i^2)} \qquad (i = a, b)$$

$$\tau_j - \sigma_{zx}(-\frac{h_a}{2}) = \sigma_{zx}(+\frac{h_b}{2})$$

$$p_j = \sigma_z(-\frac{h_a}{2}) = \sigma_z(+\frac{h_b}{2})$$

two useful relations are obtained:

$$D_a D^3 \beta_a - \frac{C_a \nu_a h_a}{2R_a} D\beta_a - \frac{C_a \nu_a h_b}{2R_a} D\beta_b + \frac{h_a}{2} D\tau_j - p_j - \frac{C_a \nu_a}{R_a} Du_{o_b} - \frac{C_a}{2R_a} w_b$$

$$= \alpha_{17} + \pi_1 \tag{12}$$

$$D_b D^3 \beta_b + \frac{h_b}{2} D\tau_j + p_j - \frac{C_b \nu_b}{R_b} Du_{o_b} - \frac{C_b}{R_b^2} w_b = \alpha_{27} + \pi_2 \quad (13)$$

Note that in the above, use has been made of the conditions that the laminae are bonded together and that no slippage occurs in the joints between laminae. The former is given by

$u_z(\frac{-h_a}{2}) = u_z(\frac{h_b}{2})$ which implies that $\bar{w}_a(x, \frac{-h_a}{2}) + w_a = \bar{w}_b(x, \frac{h_b}{2}) + w_b$

$$\begin{aligned} \text{or } w_a &= w_b + \bar{w}_b(x, \frac{h_b}{2}) - \bar{w}_a(x, \frac{-h_a}{2}) \\ &= w_b + \hat{E} \quad (14) \end{aligned}$$

where $\hat{E} = \bar{w}_b(x, \frac{h_b}{2}) - \bar{w}_a(x, -\frac{h_a}{2})$.

The latter condition is expressed by

$$u_x(\frac{-h_a}{2}) = u_x(\frac{h_b}{2})$$

$$u_{o_a} - \frac{h_a}{2}\beta_a = u_{o_b} + \frac{h_b}{2}\beta_b$$

$$u_{o_a} = u_{o_b} + (\frac{h_a}{2}\beta_a + \frac{h_b}{2}\beta_b) \quad (15)$$

Making use of (11), (14) and (15) in the first integrated equilibrium equation, two further relations are obtained:

$$\frac{C_a h_a}{2} D^2\beta_a + \frac{C_a h_b}{2} D^2\beta_b - \tau_j + C_a D^2 u_{o_b} + \frac{C_a \nu_a}{R_a} Dw_b = \alpha_{37} + \pi_3 \quad (16)$$

$$\tau_j + C_b D^2 u_{o_b} + \frac{C_b \nu_b}{R_b} Dw_b = \alpha_{47} + \pi_4 \quad (17)$$

Using (11A) and the second of the integrated equilibrium equations together with (14) and (15), two final relations are obtained:

$$D_a D^2\beta_a - \frac{5}{6} G_c^a h_a \beta_a + \frac{5}{12} h_a \tau_j - \frac{5}{6} G_c^a h_a Dw_b = \alpha_{57} + \pi_5 \quad (18)$$

$$D_b D^2\beta_b - \frac{5}{6} G_c^b h_b \beta_b + \frac{5}{12} h_b \tau_j - \frac{5}{6} G_c^b h_b Dw_b = \alpha_{67} + \pi_6 \quad (19)$$

where in the above

$$\alpha_{17} = \frac{N_{T\theta_a}}{R_a(1-\nu_a)} - \frac{C_a \hat{E}}{R_a^2} - \frac{E_a}{R_a^2(1-\nu_a^2)} \int_{-\frac{h_a}{2}}^{\frac{h_a}{2}} \bar{w}_a dz$$

$$+ D^2\{\frac{M_{Tx_a}}{1-\nu_a} - \frac{E_a \nu_a}{R_a(1-\nu_a^2)} \int_{-\frac{h_a}{2}}^{\frac{h_a}{2}} z\bar{w}_a dz\}$$

$$\alpha_{27} = \frac{N_{T\theta_b}}{R_b(1-\nu_b)} - \frac{E_b}{R_b^2(1-\nu_b{}^2)} \int_{-\frac{h_b}{2}}^{\frac{h_b}{2}} \bar{w}_b dz$$

$$+ D^2 \left\{\frac{M_{Tx_b}}{1-\nu_b} - \frac{E_b \nu_b}{R_b(1-\nu_b^2)} \int_{-\frac{h_b}{2}}^{\frac{h_b}{2}} z\bar{w}_b dz\right\}$$

$$\alpha_{37} = D \left\{\frac{N_{Tx_a}}{1-\nu_a} - \frac{C_a \nu_a}{R_a} \hat{E} - \frac{E_a \nu_a}{R_a(1-\nu_a^2)} \int_{-\frac{h_a}{2}}^{\frac{h_a}{2}} \bar{w}_a dz\right\}$$

$$\alpha_{47} = D \left\{\frac{N_{Tx_b}}{1-\nu_b} - \frac{E_b \nu_b}{R_b(1-\nu_b^2)} \int_{-\frac{h_b}{2}}^{\frac{h_b}{2}} \bar{w}_b dz\right\}$$

$$\alpha_{57} = \Delta_a + D \left\{\hat{E} + \frac{M_{Tx_a}}{1-\nu_a} - \frac{E_a \nu_a}{R_a(1-\nu_a^2)} \int_{-\frac{h_a}{2}}^{\frac{h_a}{2}} z\bar{w}_a dz\right\}$$

$$\alpha_{67} = \Delta_b + D \left\{\frac{M_{Tx_b}}{1-\nu_b} - \frac{E_b \nu_b}{R_b(1-\nu_b^2)} \int_{-\frac{h_b}{2}}^{\frac{h_b}{2}} z\bar{w}_b dz\right\} \qquad (20)$$

with

$$\pi_1 = -p_{1a} - \frac{h_a}{2} D\tau_{1a}$$

$$\pi_2 = +p_{2b} - \frac{h_b}{2} D\tau_{2b}$$

$$\pi_3 = -\tau_{1a}$$

$$\pi_4 = +\tau_{2b}$$

$$\pi_5 = -\frac{5}{12} h_a \tau_{1a}$$

$$\pi_6 = -\frac{5}{12} h_b \tau_{2b} \qquad (21)$$

$$\Delta_i = \frac{5}{4}\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \left\{ G_c^i \bar{w}_i' - \left(\frac{2z}{h_i}\right)^2 \left[G_c^i \bar{w}_i' - \frac{1}{R}\int_{-\frac{h_i}{2}}^{z} \Omega dy \right] \right\} dz \qquad (i= a,\ b)$$

The governing equations may be transformed to a more useful form for solution. Since they are cumbersome for manipulation in their present form, a matrix notation was used to define the coefficients of the unknowns. These are tabulated in Appendix C. After some manipulation of the governing equations, the following are obtained:

$$(g_1 D^7 + g_2 D^5 + g_3 D^3 + g_4 D)\, w_b = L_I(x) \tag{22}$$

$$(b_{11}D^4 + b_{12}D^2 + b_{13})\,\beta_b = L_{II}(x) - (b_{14}D^3 + b_{15}D)\, w_b \tag{23}$$

$$a_{23}D\beta_a^2 + L_{III}(x) - (a_{21}D^2 + a_{22})\,\beta_b - a_{24}Dw_b \tag{24}$$

$$D^2u_{ob} = L_{IV}(x) - k_1D^2\beta_a - k_2D\,\beta_b - k_3Dw_b \tag{25}$$

$$\tau_j = L_V(x) - k_4D^2u_{ob} - k_5Dw_b \tag{26}$$

$$P_j = L_{VI}(x) - k_6D^3\beta_b - k_7D\tau_j + k_8Du_{ob} + k_9w_b \tag{27}$$

III. SOLUTION OF GOVERNING EQUATIONS

Homogeneous Solution:

Consider first W_b. Assuming a solution of the form $W_b = e^{sx}$ and letting $y = s^2$, (22) then takes the form

$$y^3 + \frac{g_2}{g_1} y^2 + \frac{g_3}{g_1} y + \frac{g_4}{g_1} = 0 \tag{28}$$

for the homogeneous solution.

The solution of equations in the form (28) is developed in reference (42) and leads to three possibilities for the roots:

Case 1: there are two conjugate imaginary roots and one real root

Case 2: there are three real and unequal roots

Case 3: there are three real roots of which at least two are equal.

Case 1 leads to a solution of (22) which is of the form

$$W_{bH} = V_1 e^{c_1 x} + V_2 e^{-c_1 x} + e^{c_2 x}(V_3 \cos c_3 x + V_4 \sin c_3 x) + e^{-c_2 x}(V_5 \cos c_3 x + V_6 \sin c_3 x) \tag{29}$$

where $V_1 - V_6$ are constants to be evaluated through boundary conditions. The constants $c_1 - c_3$ are defined in Appendix C.

The Case 2 solution can take on several forms depending on wether the roots of (28) are positive or negative. The final forms

for the case of one, two and three positive real roots respectively, are:

$$W_{bh} = V_1e^{c_4x} + V_2e^{-c_4x} + V_3 \cos c_5x + V_4 \sin c_5x$$
$$+ V_5 \cos c_6x + V_6 \sin c_6x \quad (30)$$

$$W_{bH} = V_1e^{c_4x} + V_2e^{-c_4x} + V_3e^{c_5x} + V_4e^{-c_5x}$$
$$+ V_5 \cos c_6x + V_6 \sin c_6x \quad (31)$$

$$W_{bH} = V_1e^{c_4x} + V_2e^{-c_4x} + V_3e^{c_5x} + V_4e^{-c_5x}$$
$$+ V_5e^{c_6x} + V_6e^{-c_6x} \quad (32)$$

where, again, the $V_1 - V_6$ are boundary value constants and $c_4 - c_6$ are defined in Appendix C.

Case 3 represents the degenerate forms (30) - (32) where two of the roots are equal. The equations then take the form:

$$W_{bH} = V_1e^{c_4x} + V_2e^{-c_4x} + (V_3 + V_5X) \cos c_5x + (V_4 + V_6x)\sin c_5x \quad (33)$$

$$W_{bH} = (V_1 + V_3x)e^{c_4x} + (V_2 + V_4x)e^{-c_4x} + V_5 \cos c_6x$$
$$+ V_6 \sin c_6x \quad (34)$$

$$W_{bH} = (V_1 + V_3x)e^{c_4x} + (V_2 + V_4x)e^{-c_4x} + V_5e^{c_6x}$$
$$+ V_6e^{-c_6x}. \quad (35)$$

In treating single-layered cylinders, it is possible to write the $c_1 - c_6$ explicitly in terms of the physical quantities

involved and thus have a feel for the physical behavior of the shell. In the present derivation, however, these expressions are so lengthy and involved that, unless one has a specific example in mind, their explicit form for the general case is of dubious utility. A point is reached where one must decide whether to obscure the physical situation with mathematics or obscure the mathematics with the physical quantities involved. The former course was chosen to show where the mathematical formulation is analogous to that for the single-layered cylinder (Case 1) and where it diverges (Cases 2 and 3). To import greater physical significance to the equations it becomes necessary to either consider a specific case and determine the form and constants listed in Appendix C will take or resort to numerical evaluation of the constants.

Consider next equation (25). This may be integrated directly to give

$$U_{ob} = \iint L_{IV}(x)\,dxdx - k_1\beta_a - k_2\beta_b - k_3\int w_b dx + V_7 x + V_8 \qquad (36)$$

where V_7 and V_8 are also boundary value constants. Since there are also eight boundary conditions, four at each edge, it follows that the homogeneous solutions for the remaining unknowns β_a and β_b are not required. Their particular solutions will suffice to satisfy the governing equations. This is equivalent to setting the boundary value constants of the β_a and β_b homogeneous solutions to zero.

Total Solution:

The total solution will consist of the homogeneous solutions given in the previous section together with whatever particular solutions are called for due to the mechanical and thermal loading for any given problem. Total solutions for displacements will take the form:

$$W_b(x) = W_{bH} + W_{b\ \underline{part}} \qquad (37)$$

$$\beta_b(x) = \beta_{b\ \underline{part}} \qquad (38)$$

$$\beta_a(x) = \beta_{a\ \underline{part}} \qquad (39)$$

where $W_{b\ \underline{part}}$ is the particular solution of (22) due to $L_I(x)$, and

$\beta_{b\ \underline{part}}$ is the particular solution of (23) due to $L_{II}(x) - (b_{14}D^3 + b_{15}D)W_b(x)$, and

$\beta_{a\ \underline{part}}$ is the particular solution of (24) due to $L_{III}(x) - (a_{21}D^2 + a_{22})\beta_b(x) - a_{24}DW_b(x)$.

Once (37) - (39) are known, u_{ob} may be found from (36) and the joint shear and normal stresses from (26) and (27) respectively.

IV. BOUNDARY CONDITIONS

Boundary conditions for plates and shells are listed in many sources (21), (34), (37), (36), (37). For the axi-symmetric case, they are usually stated as:

At the edges $x = o$ and $x = L$

either u or N prescribed

either w or Q prescribed (40)

either β or M prescribed

For multilayered problems, the same boundary conditions usually apply if N, M and Q are interpreted as resultants $\overline{N}$, $\overline{M}$, $\overline{Q}$. Considering a two-layered cylinder for example, laminas a and b, the resultants would be:

$$\overline{N} = \overline{N}_a + \overline{N}_b$$

$$\overline{Q} = Q_a + Q_b \qquad (41)$$

$$M = M_a + M_b + \left(\frac{h_a}{2} + \frac{h_b}{2}\right)N_a$$

However, (40), (41) do not make use of the no slip - no delamination conditions. These provide two constraints not only on displacements but on boundary condtions as well.

Appropriate boundary conditions can be derived using the principle of virtual displacements of the layer middle surfaces.

$$U_i^* \qquad \beta_i^* \quad (i = a,b) \qquad W_i^* \tag{42}$$

where the asterisk denotes a virtual quantity. Multiplying equations (10) by the virtual displacements (42) in the order listed, adding the products, integrating over shell length and summing over the layers, we get:

$$\sum_{i=a}^{b} \int_C \{ [N_{x_i} + (\tau_{1i} - \tau_{2i})] u_i^* +$$

$$[M_{x_i} - Q_i + 1/2\, hi\, (\tau_{1i} + \tau_{2i})]\, \beta_i^* + \tag{43}$$

$$[Q_i - (1/R) N_{\theta i} + (P_{1i} - P_{2i})]\, w_i^* \}\, dC] = 0$$

since the virtual work done by a shell in equilibrium through a virtual displacement is equal to zero.

After integration by parts, the virtual work principle for the multilayered cylinder takes the form:

$$\sum_{i=a}^{b} \{ [N_{x_i} u_i^* + M_{x_i} \beta_i^* + Q_i W_i^*]_0^L$$

$$+ \int_0^L [(\tau_{1i} - \tau_{2i}) u_i^* + 1/2\, hi\, (\tau_{1i} + \tau_{2i})\, \beta_i^*$$

$$+(p_{1i} - p_{2i})\, w_i^* \,]\, dx \,\} =$$

$$\sum_{i=a}^{b} \int_0^L \Big[N_{x_i} U_i^* + M_{x_i}\, \beta_i^* + Q_i(\beta_i^* + W_i^*) \qquad (44)$$

$$+ \frac{N_{\theta i}}{R}\, w_i\Big]\, dx$$

The first quantity in brackets on the left hand side of (44) involves terms which are specified at the boundaries, namely:

$$\sum_{i=a}^{b} [N_{x_i} u_i^* + M_{x_i}\, \beta_i^* + Q_i w_i^*]_0^L$$

Summing over layers gives

$$[N_{x_a} u_a^* + M_{x_a}\, \beta_a^* + Q_a w_a^* + N_{x_b} u_b^* + M_{x_b}\, \beta_b^* + Q_b w_b^*]_0^L$$

Since this derivation does not allow for slip or delamination, the virtual displacements must be constrained to:

$$u_a^* = u_b^* + (1/2)h_a\, \beta_a^* + (1/2)h_b\, \beta_b^* \qquad (45)$$

$$w_a^* = w_b^* + (\overline{w}_b - \overline{w}_a)^*$$

Substituting into (49) and collecting terms, the quantities to be specified at x = o and x = L are found to be:

$$
\begin{aligned}
&\text{either } N_{x_a} + N_{x_b} \quad \text{or} \quad u_b \text{ specified} \\
&\text{either } 1/2\, h_a N_{x_a} + M_{x_a} \quad \text{or} \quad \beta_a \text{ specified} \\
&\text{either } 1/2\, h_b N_{x_a} + M_{x_b} \quad \text{or} \quad \beta_b \text{ specified} \\
&\text{either } Q_a + Q_b \text{ or } w_b \text{ specified} \\
&\qquad Q_a (\bar{w}_b - \bar{w}_a) \text{ specified}
\end{aligned}
\qquad (46)
$$

Note that the first four conditions are not a unique set. Other possibilities are (u_a, u_b, β_a, w_b) or (u_a, u_b, β_b, w_b) specified, but the set (46) seems to be a good choice.

The last condition of (46) is the result of retaining thermal expansion through-the-thickness while dropping terms of order h/R throughout the remainder of the derivation. For cases where $\bar{w}$ must be retained, the following procedure can be employed. For simplicity, free-free boundary conditions are considered, though the treatment is analogous for any set of conditions specified.

Since the last of (46) is a temperature dependent, $\bar{w}_b - \bar{w}_a$

is a priori specified. Hence, for free boundaries, either Qa = 0 or $(\bar{w}_b - \bar{w}_a) = 0$. Qa = 0 is acceptable for special cases but is not generally true. $(\bar{w}_b - \bar{w}_a\ )$ can be rigorously satisfied by redefining the reference surface location in layer a, i.e.,

$$\bar{w}_a\ (x,\ Z_o) = \int_o^{Z_o} \alpha_{c_a} T_a\ (x,z)\ dz$$

$$w_b\ (x,\ \frac{h_b}{2}\ ) = \int_o^{h_b/2} \alpha_{c_b} T_b\ (x,\ z)\ dz$$

(47)
([redacted])

Under these conditions, the no-slip requirement becomes

$$u_{o_a} = u_{o_b} + (-Z_o\ \beta_a + 1/2\ h_b\beta_b)$$

(48)
([redacted])

and all other results remain the same if 1/2 ha is replaced by $-Z_o$.

Note that when this approach is used, the second strain definition of equation (8) must be used in the form:

$$\epsilon_\theta = \frac{w_i + \bar{w}_i}{R_i + Z} \qquad (i = a,b)$$

in order to have a zero stress state for the case of an isotropic material with the same α_c in both layers and T = constant.

V. SAMPLE PROBLEM

The problem selected was the slow cooling of pyrolytic graphite. Because of the difference in thermal expansion coefficients in both "a" and "c" directions of the pyrolytic graphite and mandrel material and also because of curvature effects, normal and shear stresses at the deposit-mandrel interface will be formed during the cooling process. These may be of sufficient magnitude to cause flaking or delamination. An investigation of their behavior with changing material and geometric properties is therefore of interest.

The test case considered a laminated cylinder with free-free edges and a constant temperature T_o = -1000°F. The material properties used were averaged in the range 3000°F - 2000°F. Layer a, the top-most layer, was taken to be pyrolytic graphite, layer b commercial (ATJ) graphite. The properties used are given in Table (B.1). The calculations were performed with the aid of a CDC 6600 computer. The program tabulation is given in Appendix A. Results showing the behavior of τ_j and p_j due to variation in lamina thickness and/or E/G_c ratio are in Appendix B. From these, the following conclusions may be drawn:

1. Figures (B.1) and (B.2) indicate a decrease in normal and shear stresses at the mandrel - PG interface as the E/G_c ratio is increases, implying that a material weak in shear is desirable to minimize stresses and therefore the possibility of debonding.

2. Figures (B.3) and (B.4) show results when $h = 0.50$, $E/G_c = 20$ and $5 \geq h_a/h_b \geq 1$. Figures (B.5) and (B.6) show the case where $h_b = 0.25$, $E/G_c = 20$, $4 \geq h_a/h_b \geq 1$. All the curves indicate that a high h_a/h_b ratio is the desirable, for given material properties, for minimization of normal and shear stresses at the joint. This implies that a thin graphite mandrel is preferable to a thick mandrel. Note that when $h_a/h_b = 1$, the ultimate stress ($\sigma_{ult} = 18{,}000$ psi) in tension for pyrolytic graphite is exceeded.

3. To determine the behavior of the roots of equation (22) and establish a range wherein the various forms of solution (29)-(35) will occur, computations were also made for $1000 \geq h_a/h_b \geq 0.001$ with E/G_c at 50, 20 and 2.6. Results show that the Case 1 solution in the form (29) occurs whenever $h_a/h_b \geq 1$ independent of the E/G_c ratio. The constants c_1 - c_3 are affected by E/G_c, the constant c_1 being much more sensitive to a change in E/G_c than c_2 or c_3.

4. The first two terms of (29) have their principal effects at the edge only. It was also observed that for $h_a/h_b >> 1$, the value c_1 becomes so very large that the boundary value constants V_1 and V_2 tend to become very small, and as a first appromation the terms containing them can be dropped from the solution functions. In this case, (29) takes on a form similar to that for the solution for radial displacement of a single-layered cylinder, with suitably defined constants.

5. It should be kept in mind that all the above remarks are based on the numerical results and are valid in the range.

$1000 \gg h_a/h_b \gg 0.001$; $400 \gg L/h \gg 26$; $0.05 \gg h/R \gg 0.0033$

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APPENDIX A
Computer Program

```
FORTRAN IV PROGRAM PGE(INPUT,OUTPUT,TAPE5=INPUT,TAPE9=OUTPUT)      ***** 1
      DIMENSION ICO(6), IIM(6), IRE(6)                              PGE   2
      DIMENSION ORQ(10), HP(10), HQ(10), COEF(5), G3ACO(5), ROOTS(2,6)  PGE   3
      COMMON /SHEAR/ QA,QB,FQ                                       PGE   4
      COMMON /KLP/ IC1,IC2,IC3,IC4,IC5,IC6,IM1,IM2,IM3,IM4,IM5,IM6,IR1,IPGE   5
     1R2,IR3,IR4,IR5,IR6                                            PGE   6
      COMMON DLTEMP                                                 PGE   7
      COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,PGE   8
     1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),BT(30),FNTOA,FNPGE   9
     2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FMPGE  10
     3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DWPGE  11
     4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DWPGE  12
     5UA                                                            PGE  13
      EQUIVALENCE (IC1,ICO(1)), (IM1,IIM(1)), (IR1,IRE(1))          PGE  14E
      REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2                     PGE  15
      READ (5,12) (ORQ(I),I=1,4)                                    PGE  16R
      READ (5,12) (HP(I),I=1,6)                                     PGE  17R
      READ (5,12) (HQ(M),M=1,4)                                     PGE  18R
      READ (5,11) TEMP,DLTEMP                                       PGE  19R
      READ (5,11) EA,EAC,NUA,NUAC                                   PGE  20R
      READ (5,11) EB,EBC,NUB,NUBC                                   PGE  21R
      READ (5,9) BT(1),BT(2),BT(3),BT(4)                            PGE  22R
      READ (5,10) (DELPH(J),J=1,11)                                 PGE  23R
      WRITE (9,13) (HQ(M),M=1,4)                                    PGE  24W
      WRITE (9,14) TEMP                                             PGE  25W
      WRITE (9,15) EA,EAC,NUA,NUAC                                  PGE  26W
      WRITE (9,16) EB,EBC,NUB,NUBC                                  PGE  27W
      COEF(1)=50.                                                   PGE  28
      COEF(2)=20.                                                   PGE  29
      COEF(3)=2.6                                                   PGE  30
      DO 8 I14=1,6                                                  PGE  31
      DO 8 M=1,4                                                    PGE  32
      DO 8 L=1,3                                                    PGE  33
      HA=HP(I14)                                                    PGE  34
      HB=HQ(M)                                                      PGE  35
      R=30.                                                         PGE  36
      G3ACO(L)=EA/COEF(L)                                           PGE  37
      G3A=G3ACO(L)                                                  PGE  38
      G3B=EB/(2.*(1.+NUBC))                                         PGE  39
      CALL PRELIM (L,M,I14)                                         PGE  40
      CALL POLYR (6,G,ROOTS,D)                                      PGE  41
      CALL RICF (ROOTS,AC,KLIP)                                     PGE  42
      CALL THERM                                                    PGE  43
      CALL MISC                                                     PGE  44
      DO 7 IJP=1,4                                                  PGE  45
      ELL=ORQ(IJP)                                                  PGE  46
      IF (ELL.EQ.0.0) GO TO 7                                       PGE  47
      AM(1)=AC(2)**2-AC(3)**2                                       PGE  48
      AM(2)=2.*(AC(2)*AC(3))                                        PGE  49
      AM(3)=AC(2)*AM(1)-AC(3)*AM(2)                                 PGE  50
      AM(4)=AC(3)*AM(1)+AC(2)*AM(2)                                 PGE  51
      AM(17)=AM(1)*AC(3)-AM(2)*AC(2)                                PGE  52
      AM(18)=AM(1)*AC(2)+AM(2)*AC(3)                                PGE  53
      AM(25)=AM(1)*AC(2)+AM(2)*AC(3)                                PGE  54
      AM(26)=AM(1)*AC(3)-AM(2)*AC(2)                                PGE  55
      AM(5)=AC(2)*AM(3)-AC(3)*AM(4)                                 PGE  56
      AM(6)=AC(3)*AM(3)+AC(2)*AM(4)                                 PGE  57
      IF (KLIP-5) 1,2,3                                             PGE  58
    1 CONTINUE                                                      PGE  59
      IF (KLIP-2) 4,5,6                                             PGE  60
```

```
 2 CALL EKLIPS                                                         PGE   61
   GO TO 6                                                             PGE   62
 3 CALL EKLIP6                                                         PGE   63
   GO TO 6                                                             PGE   64
 4 CALL EKLIP1                                                         PGE   65
   GO TO 6                                                             PGE   66
 5 CALL EKLIP2                                                         PGE   67
 6 CONTINUE                                                            PGE   68
   CALL MISC                                                           PGE   69
 7 CONTINUE                                                            PGE   70
 8 CONTINUE                                                            PGE   71
                                                                       PGE   72
                                                                       PGE   73
                                                                       PGE   74
 9 FORMAT (4E15.7)                                                     PGE   75
10 FORMAT (6E12.5)                                                     PGE   76
11 FORMAT (4E15.0)                                                     PGE   77
12 FORMAT (16F4.0)                                                     PGE   78
13 FORMAT (4H HB=12F9.3)                                               PGE   79
14 FORMAT (1X,5HTEMP=E12.5)                                            PGE   80
15 FORMAT (1X,6HAPROP=4E15.7)                                          PGE   81
16 FORMAT (1X,6HBPROP=4E15.7/)                                         PGE   82
   END                                                                 PGE   83-
   SUBROUTINE UCOFF (FNTWW,CONST,X)                                    ***** 1
   COMMON DLTEMP                                                             2
   COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,        3
  1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),BT(30),FNTOA,FN        4
  2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FM        5
  3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DW        6
  4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DW        7
  5UA                                                                       8
   REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2                                 9
   PP4=-(ELL/2.)*AC(1)                                                      10
   PP5=-(ELL/2.)*AC(2)                                                      11
   PP6=-(ELL/2.)*AC(3)                                                      12
   CONST=BDE(19)*WA+BDE(20)*WB+BDE(21)*FNTWW-BT(20)*X/CAPCB                 13
   RETURN                                                                   14
   END                                                                      15-
   SUBROUTINE PRELIM (L,M,I14)                                         ***** 1
   COMMON DLTEMP                                                       PRELI 2
   COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,PRELI 3
  1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),BT(30),FNTOA,FNPRELI 4
  2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FMPRELI 5
  3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DWPRELI 6
  4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DWPRELI 7
  5UA                                                                  PRELI 8
   REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2                           PRELI 9
   CA=EA*HA/(1.-NUA**2)                                                PRELI10
   CB=EB*HB/(1.-NUB**2)                                                PRELI11
   DA=EA*(HA**3)/(12.*(1.-NUA**2))                                     PRELI12
   DB=EB*(HB**3)/(12.*(1.-NUB**2))                                     PRELI13
   H=HA+HB                                                             PRELI14
   CACB=CA*CB                                                          PRELI15
   CAPCB=CA+CB                                                         PRELI16
   NUNU=NUA-NUB                                                        PRELI17
   GAMA=(5./6.)*(G3A*HA)                                               PRELI18
   GAMB=(5./6.)*(G3B*HB)                                               PRELI19
   K(15)=1.+CB/CA                                                      PRELI20
   K(16)=1.+CA/CB                                                      PRELI21
   K(17)=HA/HB                                                         PRELI22
```

```
A(11)=(1./12.)*(HA**2)+H*HA/(4.*K(16))
A(12)=-HA*NUNU/((2.*R)*K(16))
A(13)=(HB*HB)*(K(15)/K(16))/12.+H*HB/(4.*K(16))
A(14)=HB*A(12)/HA
A(15)=-(H/HA)*A(12)
FDUM=(CA*NUA+CB*NUB)**2
CDUM=CAPCB**2
BDUM=(CA*CAPCB)*(R**2)
A(16)=(FDUM-CDUM)/BDUM
A(21)=(HB**2)/12.+(5./24.)*(HB**2)/K(15)
A(22)=-(5./6.)*((G3B/EB)*(1.-NUB**2))
A(23)=(5./24.)*(HA*HB)/K(15)
A(24)=-((5./6.)*HB/HA)*A(12)-A(22)
A(31)=((1./12.)*(HA**2))*(1.+5./(2.*K(16)))
A(32)=-(5./6.)*((G3A/EA)*(1.-NUA**2))
A(33)=(K(15)/K(16))*A(23)
A(34)=(5./12.)*(HA/(R*K(16)))-GAMA/CA
   A(41) = B(11)                     A(51)=B(21)
   A(42) = B(12)                     A(52) = B(22)
   A(43) = B(13)                     A(53) = B(23)
   A(44) = B(14)                     A(54) = B(24)
   A(45) = B(15)                     A(55) = B(25)

A(41)=A(13)*A(23)-A(11)*A(21)
A(42)=A(14)*A(23)-A(11)*A(22)-A(21)*A(12)
A(43)=-A(12)*A(22)
A(44)=A(15)*A(23)-A(11)*A(24)
A(45)=A(16)*A(23)-A(12)*A(24)
A(51)=A(23)**2-A(31)*A(21)
A(52)=-A(31)*A(22)-A(32)*A(21)
A(53)=-A(32)*A(22)
A(54)=A(23)*A(34)-A(31)*A(24)
A(55)=-A(32)*A(24)
   G(1)  =  D(1)
   G(3)  =  D(2)
   G(5)  =  D(3)
   G(7)  =  D(4)
G(1)=A(51)*A(44)-A(41)*A(54)
G(3)=A(51)*A(45)+A(52)*A(44)-A(42)*A(54)-A(41)*A(55)
G(5)=A(52)*A(45)+A(53)*A(44)-A(42)*A(55)-A(43)*A(54)
G(7)=A(53)*A(45)-A(43)*A(55)
G(2)=0.0
G(4)=G(2)
G(6)=G(4)
WRITE (9,3) L,G3A,G3B
WRITE (9,4) HA,HB,R,TEMP
WRITE (9,5) (IOP,G(IOP),IOP=1,7)
WRITE (9,1) CA,CB,DA,DB
WRITE (9,6) CACB,CAPCB,GAMA,GAMB
WRITE (9,8) HA,HB,ELL,L,K(17)
WRITE (9,7)
K(1)=1.0
K(2)=G(3)/G(1)
K(3)=G(5)/G(1)
K(4)=G(7)/G(1)
FRUMP=(3.*K(3)-K(2)**2)/3.
HRMP=(2.*(K(2)**3)-9.*(K(2)*K(3))+27.*K(4))/27.
   FRUMP = SMALL A
   HRMP = SMALL B
RATIO=HA/HB
```

```
      DISC=(HRPP**2)/4.+(FRUMP**3)/27.
      WRITE (9,2) HA,HB,RATIO,DISC
C        DISCRIMINANT.LT. 0     IMPLIES KLIP = 2,3,4, OR 5 DEPENDING ON R
C        DISCRIMINANT.EQ.0    IMPLIES DEGENERATE CASE
C        DISCRIMINANT.GT.0      KLIP = 1       MODIFIED CLASSICAL SOLUTION
      RETURN
C
C
    1 FORMAT (1X,3HCA=E12.5,1X,3HCB=E12.5,1X,3HDA=E12.5,1X,3HDB=E12.5/)
    2 FORMAT (//1X,3HHA=F10.3,1X,3HHB=F10.3,5X,6HHA/HB=F15.7,5X,13HDISCR
     1IMINANT=E15.7//)
    3 FORMAT (2X,4HG3A(I2,2H)=E15.7,4HG3B E15.7/)
    4 FORMAT (2X,4H HA=F10.2,2X,4H HB=F10.2,2X,2HR=F10.2,2X,6H TEMP=F10.
     12/)
    5 FORMAT (4(2X,2HG(I2,2H)=E12.5)/3(2X,2HG(I2,2H)=E12.5))
    6 FORMAT (2X,5HCACB=E12.5,1X,6HCAPCB=E12.5,1X,5HGAMA=E12.5,1X,5HGAMB
     1=E12.5/)
    7 FORMAT (20H  A(I) ARE            )
    8 FORMAT (F12.5,F12.5,F12.5,I3,1X,7H HA/HB=F12.5)
      END
      SUBROUTINE MISC
      COMMON OLTEMP
      COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,
     1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),HT(30),FNTOA,FN
     2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FM
     3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DW
     4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DW
     5UA
      REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2
      BDE(19)=CA*HA/(2.*CAPCB)
      BDE(20)=(HB/HA)*BDE(19)
      BDE(21)=(1./R)*(CA*NUA+CB*NUB)/CAPCB
      BDE(22)=CB
      BDE(23)=BDE(22)*NUB/R
      BDE(24)=(HB**2)*BDE(22)/12.
      BDE(25)=HB/2.
      BDE(26)=BDE(22)/(R**2)
C
          K(1)=BDE(19)
          K(2)=BDE(20)
          K(3)=BDE(21)
          K(4)=BDE(22)
          K(5)=K(8)=BDE(23)
          K(6)=BDE(24)
          K(7)=BDE(25)
          K(9)=BDE(26)


      AM(24)=AC(2)**2+AC(3)**2
      BDE(37)=GAMA+GAMB
      BDE(38)=BDE(37)-(H/12.)*BDE(23)
      LOAD2=BT(11)
      LOAD1=0.0


      BDE(61)=BDE(19)*BDE(22)
      BDE(62)=BDE(20)*BDE(22)
      BDE(63)=BDE(21)*BDE(22)-BDE(23)
      BDE(64)=BDE(24)+BDE(25)*(BDE(22)*BDE(20))
      BDE(65)=BDE(25)*(BDE(22)*BDE(19))
```

```
      BDE(66)=BDE(25)*BDE(63)
      BDE(67)=BDE(23)*BDE(19)
      BDE(68)=BDE(23)*BDE(20)
      BDE(69)=BDE(26)-BDE(23)*BDE(21)
      AM(32)=(CA*NUA+CB*NUB)/R
      AM(33)=CA*HA/2.
      AM(34)=CA*HB/2.
      AM(35)=DA+(H*HA)*CA/4.
      AM(36)=DB+(H*HB)*CA/4.
      AM(37)=AM(33)+AM(34)
      AM(38)=AM(37)*NUA/R
      AM(39)=GAMA+GAMB
      AM(42)=DA+(CA/4.)*(HA**2)
      AM(43)=(HA*HB)*(CA/4.)
      BDE(81)=AM(33)*NUA/R
      BDE(82)=AM(34)*NUA/R
      AM(44)=AM(42)-AM(33)*BDE(19)
      AM(45)=AM(43)-AM(33)*BDE(20)
      AM(46)=BDE(81)-AM(33)*BDE(21)
      AM(47)=DB+(CA/4.)*(HB**2)
      AM(48)=AM(43)-AM(34)*BDE(19)
      AM(49)=AM(47)-AM(34)*BDE(20)
      AM(50)=BDE(82)-AM(34)*BDE(21)
      BDE(83)=(H/12.)*(BDE(22)*BDE(19))
      BDE(84)=(H/12.)*(BDE(22)*BDE(20))
      BDE(85)=AM(39)+(H/12.)*(BDE(22)*BDE(21)-BDE(23))
      BDE(71)=AM(33)-CA*BDE(19)
      BDE(72)=AM(34)-CA*BDE(20)
      BDE(73)=CA*(NUA/R-BDE(21))
      BDE(74)=CA*(NUA*BT(9)/R+BT(20)/CAPCB)-BT(5)/(1.-NUA)
      BDE(75)=-CB*BDE(19)
      BDE(76)=-CB*BDE(20)
      BDE(77)=CB*(NUB/R-BDE(21))
      BDE(91)=AM(44)+AM(48)
      BDE(92)=AM(45)+AM(49)
      BDE(93)=AM(46)+AM(50)

      RETURN
      END
      SUBROUTINE THERM
      COMMON DLTEMP
      COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,
     1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),BT(30),FNTOA,FN
     2TOB,BDE(100),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FM
     3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DW
     4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DW
     5UA
      REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2
C     BT(1) = ALPHA(A)
C     BT(2) = ALPHA(AC)
C     BT(3) = ALPHA(B)
C     BT(4) = ALPHA(BC)
C     BT(5) = NTXA = NTOA
C     BT(6) = NTXB = NTOB
C     BT(7) = W(BAR-A) AT Z = -HA/2.
C     BT(8) = W(BAR-B) AT Z = +HB/2.
C     BT(9) = E(BAR) = W(BAR-B) - W(BAR-A)
C     BT(10) = ALPHA(1,7)
C     BT(11) = ALPHA(2,7)
C     BT(12) = MTXA
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      BT(13) = NTXB                                                     THERM22
      BT(14) = EXPANSION IN THICKNESS DIRECTION - A                     THERM23
      BT(15) = EXPANSION IN THICKNESS DIRECTION - B                     THERM24
      BT(16) =  MODIFIED AXIAL THERMAL RESULTANT A                      THERM25
      BT(17) = AXIAL THERMAL RESULTANT (MODIFIED) - B                   THERM26
      BT(18) =  BT(14)                                                  THERM27
      BT(19) = BT(15)                                                   THERM28
      BT(5)=(EA*HA)*(TEMP*BT(1))                                        THERM29
      BT(6)=(EB*HB)*(TEMP*BT(2))                                        THERM30
      BT(7)=-BT(2)*(TEMP*HA/2.)+DLTEMP*(BT(2)*HA/8.)                    THERM31
      BT(8)=BT(4)*(TEMP*HB/2.)+DLTEMP*(BT(4)*HB/8.)                     THERM32
      BT(9)=BT(8)-BT(7)                                                 THERM33
      BT(10)=BT(5)/(R*(1.-NUA))-CA*BT(9)/(R**2)-(DA/(2.*HA))*(DLTEMP/(R*THERM34
     1*2))*BT(2)                                                        THERM35
      BT(11)=BT(6)/(R*(1.-NUB))-(DB/(2.*HB))*(DLTEMP/(R**2))*BT(4)      THERM36
      BT(12)=(TEMP*BT(1))*(EA*HA**3)/(12.*R)+(EA*HA**3)*(DLTEMP*BT(1))/(THERM37
     112.*HA)                                                           THERM38
      BT(13)=(TEMP*BT(3))*(EB*HB**3)/(12.*R)+(EB*HB**3)*(DLTEMP*BT(3))/(THERM39
     112.*HB)                                                           THERM40
      BT(14)=(DA*NUA)*(TEMP*BT(2))/R                                    THERM41
      BT(15)=(DB*NUB)*(TEMP*BT(4))/R                                    THERM42
      BT(16)=BT(5)/(1.-NUA)-CA*(NUA*BT(9))/R-(DA*DLTEMP/(2.*HA))*(NUA*BTTHERM43
     1(2)/R)                                                            THERM44
      BT(17)=BT(6)/(1.-NUB)-(DB*DLTEMP/(2.*HB))*(NUB*BT(4)/R)           THERM45
      BT(18)=(DA*NUA/R)*(BT(2)*TEMP)                                    THERM46
      BT(19)=(DB*NUB/R)*(BT(4)*TEMP)                                    THERM47
      BT(20)=BT(16)+BT(17)                                              THERM48
      BT(21)=(DA*(DLTEMP*BT(2))/(2.*HA))*(NUA/R)                        THERM49
      BT(22)=(DB*(DLTEMP*BT(4))/(2.*HB))*(NUB/R)                        THERM50
      PP3=A(23)*A(53)                                                   THERM51
      PP10=A(32)*A(43)-A(12)*A(53)                                      THERM52
      ADUM=(PP3/CA)*(BT(10)+BT(11))                                     THERM53
      BDUM=(CA*NUA+CB*NUB)/(CA*(R*CAPCB))                               THERM54
      CDUM=PP3*(BT(16)+BT(17))                                          THERM55
      DDUM=BDUM*CDUM                                                    THERM56
      EDUM=ADUM+DDUM                                                    THERM57
      FDUM=CA*HB/(CB*CAPCB)                                             THERM58
      GDUM=(FDUM*(5./12.))*(PP10*BT(5)/(1.-NUA))                        THERM59
      HDUM=(5./12.)*(HB*PP10/CAPCB)*BT(16)                              THERM60
      PDUM=EDUM-GDUM+HDUM                                               THERM61
      QDUM=(PP10/CB)*(BT(13)-BT(18))                                    THERM62
      RDUM=PDUM+QDUM                                                    THERM63
      BDE(49)=RDUM/G(7)                                                 THERM64
      WRITE (9,1)                                                       THERM65W
      WRITE (9,4) ADUM,BDUM,CDUM,DDUM,EDUM,FDUM,GDUM,HDUM,PDUM,RDUM,BDE(THERM66W
     149)                                                               THERM67W
      WRITE (9,2)                                                       THERM68W
      WRITE (9,3) (BT(I),I=1,30)                                        THERM69W
      RETURN                                                            THERM70
                                                                        THERM71
    1 FORMAT (1X,35HCOMPONENTS OF BDE(49) ARE            /)             THERM72
    2 FORMAT (1X,10H BT ARE    )                                        THERM73
    3 FORMAT (1X,6E12.5)                                                THERM74
    4 FORMAT (1X,4E15.7)                                                THERM75
      END                                                               THERM76-
      SUBROUTINE EKLIP1                                                 ***** 1
      DIMENSION JIP(11)                                                 EKLIP 2
      COMMON /SHEAR/ QA,QB,FQ                                           EKLIP 3
      COMMON DLTEMP                                                     EKLIP 4
      COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,EKLIP 5
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1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),BT(30),FNTOA,FN
2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FM
3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DW
4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DW
5UA
 REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2
 REAL JIP
 P4=AC(1)*ELL
 P5=AC(2)*ELL
 P6=AC(3)*ELL
 BDE(1)=A(44)*(AC(1)**3)+A(45)*AC(1)
 BDE(2)=A(41)*(AC(1)**4)+A(42)*(AC(1)**2)+A(43)
 AM(30)=BDE(1)/BDE(2)
 BDE(3)=AM(5)*A(41)+AM(1)*A(42)+A(43)
 BDE(4)=AM(6)*A(41)+AM(2)*A(42)
 BDE(5)=AM(3)*A(44)+AC(2)*A(45)
 BDE(6)=AM(4)*A(44)+AC(3)*A(45)
 BDE(7)=BDE(5)*BDE(3)+BDE(6)*BDE(4)
 BDE(8)=BDE(6)*BDE(3)-BDE(5)*BDE(4)
 BDE(9)=BDE(3)**2+BDE(4)**2
 BDE(10)=BDE(7)/BDE(9)
 BDE(11)=BDE(8)/BDE(9)
 AM(7)=-A(24)*AC(1)+AM(30)*(A(21)*(AC(1)**2)+A(22))
 AM(8)=A(23)*(AC(1)**2)
 BDE(12)=AM(7)/AM(8)
 AM(19)=A(21)*AM(1)+A(22)
 AM(20)=A(21)*AM(2)
 AM(21)=A(24)*AC(2)
 AM(22)=A(24)*AC(3)
 BDE(13)=AM(19)*BDE(10)-AM(20)*BDE(11)-AM(21)
 BDE(14)=AM(19)*BDE(11)+AM(20)*BDE(10)-AM(22)
 BDE(15)=BDE(13)/A(23)
 BDE(16)=BDE(14)/A(23)
 AM(9)=AM(1)**2+AM(2)**2
 AM(10)=AM(1)*BDE(15)+AM(2)*BDE(16)
 AM(11)=AM(1)*BDE(16)-AM(2)*BDE(15)
 BDE(17)=AM(10)/AM(9)
 BDE(18)=AM(11)/AM(9)
 AM(13)=-BDE(19)*BDE(12)+BDE(20)*AM(30)-BDE(21)/AC(1)
 BDE(27)=BDE(19)*(AM(2)*BDE(18)-AM(1)*BDE(17))+BDE(20)*(AM(1)*BDE(1
10)-AM(2)*BDE(11))-BDE(21)*AC(2)
 BDE(28)=-BDE(19)*(AM(2)*BDE(17)+AM(1)*BDE(18))+BDE(20)*(AM(1)*BDE(
111)+AM(2)*BDE(10))-BDE(21)*AC(3)
 BDE(29)=AM(1)*BDE(27)+AM(2)*BDE(28)
 BDE(30)=AM(1)*BDE(28)-AM(2)*BDE(27)
 AM(14)=BDE(29)/AM(9)
 AM(15)=BDE(30)/AM(9)
 BDE(31)=EXP(P5)*COS(P6)
 BDE(32)=EXP(P5)*SIN(P6)
 BDE(33)=EXP(-P5)*COS(P6)
 BDE(34)=EXP(-P5)*SIN(P6)
 BDE(51)=AC(1)*(HA*BDE(12)-HB*AM(30))
 BDE(52)=-BDE(16)*HA+BDE(11)*HB
 BDE(53)=BDE(17)*HA-BDE(10)*HB
 BDE(54)=AC(3)*BDE(52)+AC(2)*BDE(53)
 BDE(55)=AC(2)*BDE(52)-AC(3)*BDE(53)
 AM(12)=BT(20)/CAPCB-BDE(21)*BT(9)

       .

 BDE(70)=LOAD2+BDE(23)*BT(20)/CAPCB
```

```
      BDE(78)=CB*BT(20)/CAPCB-BT(6)/(1.-NUB)                           EKLIP66
      DO 1 I=1,6                                                       EKLIP67
      V(I)=0.0                                                         EKLIP68
      DO 1 J=1,7                                                       EKLIP69
      EN(I,J)=0.0                                                      EKLIP70
    1 CONTINUE                                                         EKLIP71
      EN(1,1)=AC(1)*(AM(44)*BDE(12)-AM(45)*AM(30))+AM(46)              EKLIP72
      EN(1,2)=EN(1,1)                                                  EKLIP73
      EN(1,3)=AM(44)*(AC(2)*BDE(17)-AC(3)*BDE(18))+AM(45)*(AC(3)*BDE(11)EKLIP74
     1-AC(2)*BDE(10))+AM(46)                                           EKLIP75
      EN(1,4)=AM(44)*(AC(3)*BDE(17)+AC(2)*BDE(18))-AM(45)*(AC(3)*BDE(10)EKLIP76
     1+AC(2)*BDE(11))                                                  EKLIP77
      EN(1,5)=EN(1,3)                                                  EKLIP78
      EN(1,6)=-EN(1,4)                                                 EKLIP79
      EN(2,1)=AC(1)*(BDE(12)*AM(48)-AM(30)*AM(49))+AM(50)              EKLIP80
      EN(2,2)=EN(2,1)                                                  EKLIP81
      EN(2,3)=AM(48)*(-AC(3)*BDE(18)+AC(2)*BDE(17))+AM(49)*(AC(3)*BDE(11EKLIP82
     1)-AC(2)*BDE(10))+AM(50)                                          EKLIP83
      EN(2,4)=AM(48)*(AC(3)*BDE(17)+AC(2)*BDE(18))-AM(49)*(AC(3)*BDE(10)EKLIP84
     1+AC(2)*BDE(11))                                                  EKLIP85
      EN(2,5)=EN(2,3)                                                  EKLIP86
      EN(2,6)=-EN(2,4)                                                 EKLIP87
      EN(3,1)=BDE(85)*AC(1)-AM(30)*(BDE(84)*(AC(1)**2)+GAMB)+BDE(12)*(BDEKLIP88
     1E(83)*(AC(1)**2)+GAMA)                                           EKLIP89
      EN(3,2)=-EN(3,1)                                                 EKLIP90
      EN(3,3)=BDE(85)*AC(2)-BDE(10)*(GAMB+BDE(84)*AM(1))+BDE(11)*(BDE(84EKLIP91
     1)*AM(2))+BDE(17)*(GAMA+AM(1)*BDE(83))-BDE(18)*(BDE(83)*AM(2))    EKLIP92
      EN(3,4)=AC(3)*BDE(85)-BDE(11)*(GAMB+BDE(84)*AM(1))-BDE(10)*(BDE(84EKLIP93
     1)*AM(2))+BDE(18)*(GAMA+AM(1)*BDE(83))+BDE(17)*(BDE(83)*AM(2))    EKLIP94
      EN(3,5)=-EN(3,3)                                                 EKLIP95
      EN(3,6)=EN(3,4)                                                  EKLIP96
      EN(4,1)=EXP(P4)*EN(1,1)                                          EKLIP97
      EN(4,2)=EXP(-P4)*EN(1,2)                                         EKLIP98
      EN(4,3)=EN(1,3)*BDE(31)-EN(1,4)*BDE(32)                          EKLIP99
      EN(4,4)=EN(1,3)*BDE(32)+EN(1,4)*BDE(31)                          EKLIP
      EN(4,5)=EN(1,5)*BDE(33)-EN(1,6)*BDE(34)                          EKLIP
      EN(4,6)=EN(1,5)*BDE(34)+EN(1,6)*BDE(33)                          EKLIP
      EN(5,1)=EXP(P4)*EN(2,1)                                          EKLIP
      EN(5,2)=EXP(-P4)*EN(2,2)                                         EKLIP
      EN(5,3)=EN(2,3)*BDE(31)-EN(2,4)*BDE(32)                          EKLIP
      EN(5,4)=EN(2,3)*BDE(32)+EN(2,4)*BDE(31)                          EKLIP
      EN(5,5)=EN(2,5)*BDE(33)-EN(2,6)*BDE(34)                          EKLIP
      EN(5,6)=EN(2,5)*BDE(34)+EN(2,6)*BDE(33)                          EKLIP
      EN(6,1)=EXP(P4)*EN(3,1)                                          EKLIP
      EN(6,2)=EXP(-P4)*EN(3,2)                                         EKLIP
      EN(6,3)=EN(3,3)*BDE(31)-EN(3,4)*BDE(32)                          EKLIP
      EN(6,4)=EN(3,3)*BDE(32)+EN(3,4)*BDE(31)                          EKLIP
      EN(6,5)=EN(3,5)*BDE(33)-EN(3,6)*BDE(34)                          EKLIP
      EN(6,6)=EN(3,5)*BDE(34)+EN(3,6)*BDE(33)                          EKLIP
                                                                       EKLIP
      EN(1,7)=AM(33)*BT(20)/CAPCB+BDE(81)*BT(9)+BT(18)-(HA/2.)*(BT(5)/(1EKLIP
     1.-NUA))-BT(12)/(1.-NUA)+(HA/2.)*BT(21)+AM(45)*BDE(49)            EKLIP
      EN(2,7)=AM(34)*BT(20)/CAPCB+BDE(82)*BT(9)+BT(19)-(HB/2.)*(BT(5)/(1EKLIP
     1.-NUA))-BT(13)/(1.-NUB)+(HB/2.)*BT(22)+AM(50)*BDE(49)            EKLIP
      EN(1,7)=-EN(1,7)                                                 EKLIP
      EN(2,7)=-EN(2,7)                                                 EKLIP
      EN(3,7)=LOAD1                                                    EKLIP
      EN(4,7)=EN(1,7)                                                  EKLIP
      EN(5,7)=EN(2,7)                                                  EKLIP
      EN(6,7)=EN(3,7)                                                  EKLIP
```

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                                                                 EKLIP
      WRITE (9,3) (BDE(I),I=1,100)                               EKLIP  W
      WRITE (9,5)                                                EKLIP  W
      WRITE (9,4) (AM(J),J=1,50)                                 EKLIP  W
      WRITE (9,10)                                               EKLIP  W
      WRITE (9,6) ((EN(I,J),J=1,7),I=1,6)                        EKLIP  W
      CALL FINGLE (EN,V)                                         EKLIP
      WRITE (9,7) (I,V(I),I=1,6)                                 EKLIP  W
                                                                 EKLIP
                                                                 EKLIP
      AM(12)=BT(20)/CAPCR-BDE(21)*BDE(49)                        EKLIP
      PP1=AC(1)*ELL/2.                                           EKLIP
      PP2=AC(2)*ELL/2.                                           EKLIP
      PP3=AC(3)*ELL/2.                                           EKLIP
      CALL CHECK1 (V,AC,PP1,PP2,PP3)                             EKLIP
      WRITE (9,7) (I,V(I),I=1,6)                                 EKLIP  W
                                                                 EKLIP
      AL(1)=-AM(30)*V(1)                                         EKLIP
      AL(2)=AM(30)*V(2)                                          EKLIP
      AL(3)=-BDE(10)*V(3)-BDE(11)*V(4)                           EKLIP
      AL(4)=BDE(11)*V(3)-BDE(10)*V(4)                            EKLIP
      AL(5)=BDE(10)*V(5)-BDE(11)*V(6)                            EKLIP
      AL(6)=BDE(11)*V(5)+BDE(10)*V(6)                            EKLIP
      AL(7)=BDE(12)*V(1)                                         EKLIP
      AL(8)=-BDE(12)*V(2)                                        EKLIP
      AL(9)=BDE(17)*V(3)+BDE(18)*V(4)                            EKLIP
      AL(10)=-BDE(18)*V(3)+BDE(17)*V(4)                          EKLIP
      AL(11)=-BDE(17)*V(5)+BDE(18)*V(6)                          EKLIP
      AL(12)=-BDE(18)*V(5)-BDE(17)*V(6)                          EKLIP
                                                                 EKLIP
                                                                 EKLIP
      AL(13)=AM(13)*V(1)                                         EKLIP
      AL(14)=-AM(13)*V(2)                                        EKLIP
      AL(15)=AM(14)*V(3)+AM(15)*V(4)                             EKLIP
      AL(16)=-AM(15)*V(3)+AM(14)*V(4)                            EKLIP
      AL(17)=-AM(14)*V(5)+AM(15)*V(6)                            EKLIP
      AL(18)=-AM(15)*V(5)-AM(14)*V(6)                            EKLIP
      WRITE (9,8)                                                EKLIP  W
      WRITE (9,9) (AL(I),I=1,18)                                 EKLIP  W
                                                                 EKLIP
      DO 2 KF=1,11                                               EKLIP
      X=DELPH(KF)                                                EKLIP
      IF (X.GT.ELL) GO TO 2                                      EKLIP
      P1=AC(1)*X                                                 EKLIP
      P2=AC(2)*X                                                 EKLIP
      P3=AC(3)*X                                                 EKLIP
                                                                 EKLIP
                                                                 EKLIP
      CALL DIFF (P1,P2,P3)                                       EKLIP
                                                                 EKLIP
                                                                 EKLIP
      WW=V(1)*EXP(P1)+V(2)*EXP(-P1)+EXP(P2)*(V(3)*COS(P3)+V(4)*SIN(P3))+EKLIP
     1EXP(-P2)*(V(5)*COS(P3)+V(6)*SIN(P3))+BDE(49)               EKLIP
      DWW=V(1)*BDE(101)+V(2)*BDE(111)+V(3)*BDE(121)+V(4)*BDE(131)+V(5)*BEKLIP
     1DE(141)+V(6)*BDE(151)                                      EKLIP
      DDWW=V(1)*BDE(102)+V(2)*BDE(112)+V(3)*BDE(122)+V(4)*BDE(132)+V(5)*EKLIP
     1BDE(142)+V(6)*BDE(152)                                     EKLIP
      WB=AL(1)*EXP(P1)+AL(2)*EXP(-P1)+EXP(P2)*(AL(3)*COS(P3)+AL(4)*SIN(PEKLIP
     13))+EXP(-P2)*(AL(5)*COS(P3)+AL(6)*SIN(P3))                 EKLIP
      DWB=AL(1)*BDE(101)+AL(2)*BDE(111)+AL(3)*BDE(121)+AL(4)*BDE(131)+ALEKLIP
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1(5)*BDE(141)+AL(6)*BDE(151)                                            EKLIP
 DDWB=AL(1)*BDE(102)+AL(2)*BDE(112)+AL(3)*BDE(122)+AL(4)*BDE(132)+AEKLIP
1L(5)*BDE(142)+AL(6)*BDE(152)                                           EKLIP
 DDDWB=AL(1)*BDE(103)+AL(2)*BDE(113)+AL(3)*BDE(123)+AL(4)*BDE(133)+EKLIP
1AL(5)*BDE(143)+AL(6)*BDE(153)                                          EKLIP
 WA=AL(7)*EXP(P1)+AL(8)*EXP(-P1)+EXP(P2)*(AL(9)*COS(P3)+AL(10)*SIN(EKLIP
1P3))+EXP(-P2)*(AL(11)*COS(P3)+AL(12)*SIN(P3))                          EKLIP
 DWA=AL(7)*BDE(101)+AL(8)*BDE(111)+AL(9)*BDE(121)+AL(10)*BDE(131)+AEKLIP
1L(11)*BDE(141)+AL(12)*BDE(151)                                         EKLIP
 DDWA=AL(7)*BDE(102)+AL(8)*BDE(112)+AL(9)*BDE(122)+AL(10)*BDE(132)+EKLIP
1AL(11)*BDE(142)+AL(12)*BDE(152)                                        EKLIP
 DDDWA=AL(7)*BDE(103)+AL(8)*BDE(113)+AL(9)*BDE(123)+AL(10)*BDE(133)EKLIP
1+AL(11)*BDE(143)+AL(12)*BDE(153)                                       EKLIP
                                                                        EKLIP
 FNTWW=(1./AC(1))*(V(1)*EXP(P1)-V(2)*EXP(-P1))+(EXP(P2)/AM(24))*(V(EKLIP
13)*(AC(2)*COS(P3)+AC(3)*SIN(P3))+V(4)*(AC(2)*SIN(P3)-AC(3)*COS(P3)EKLIP
2))+(EXP(-P2)/AM(24))*(V(5)*(-AC(2)*COS(P3)+AC(3)*SIN(P3))-V(6)*(ACEKLIP
3(2)*SIN(P3)+AC(3)*COS(P3)))+BDE(49)*X                                  EKLIP
 IF (KF.EQ.1) CALL UCOFF (FNTWW,CONST,0)                                EKLIP
 WU=-BDE(19)*WA-BDE(20)*WB-BDE(21)*FNTWW+(BT(20)/CAPCB)*X+CONST         EKLIP
 DWU=-BDE(19)*DWA-BDE(20)*DWB-BDE(21)*WW+BT(20)/CAPCB                   EKLIP
 DDWU=-BDE(19)*DDWA-BDE(20)*DDWB-BDE(21)*DWW                            EKLIP
 DDDWU=-BDE(19)*DDDWA-BDE(20)*DDDWB-BDE(21)*DDWW                        EKLIP
 TAU=LOAD1-BDE(22)*DDWU-BDE(23)*DWW                                     EKLIP
 DTAU=-BDE(22)*DDDWU-BDE(23)*DDWW                                       EKLIP
 WPJ=LOAD2-BDE(24)*DDDWB-BDE(25)*DTAU+BDE(23)*DWU+BDE(26)*WW            EKLIP
 WUA=WU+HA*WA/2.+HB*WB/2.                                               EKLIP
 DWUA=DWU+HA*DWA/2.+HB*DWB/2.                                           EKLIP
 DDWUA=DDWU+HA*DDWA/2.+HB*DDWB/2.                                       EKLIP
 CALL RSLT (X)                                                          EKLIP
 CALL PRINT (X)                                                         EKLIP
2 CONTINUE                                                              EKLIP
 RETURN                                                                 EKLIP
                                                                        EKLIP
                                                                        EKLIP
3 FORMAT (1X,9H BDE ARE /(10E12.5))                                     EKLIP
4 FORMAT (10E12.5)                                                      EKLIP
5 FORMAT (20H   AM(J) ARE          )                                    EKLIP
6 FORMAT (1X,7E11.4)                                                    EKLIP
7 FORMAT (4(1X,2HV(I1,2H)=E12.5)/2(1X,2HV(I1,2H)=E12.5))                EKLIP
8 FORMAT (1X,20H AL(I) ARE            )                                 EKLIP
9 FORMAT (6E12.5)                                                       EKLIP
10 FORMAT (21H EN(I,J)BY ROWS ARE  )                                    EKLIP
 END                                                                    EKLIP   -
 SUBROUTINE EKLIP2                                                      ***** 1
 DIMENSION JIP(11)                                                      EKLIP 2
 COMMON /SHEAR/ QA,QB,FQ                                                EKLIP 3
 COMMON DLTEMP                                                          EKLIP 4
 COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,EKLIP 5
1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),BT(30),FNTOA,FNEKLIP 6
2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FMEKLIP 7
3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DWEKLIP 8
4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DWEKLIP 9
5UA                                                                     EKLIP10
 REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2                              EKLIP11
 REAL JIP                                                               EKLIP12
 P4=AC(1)*ELL                                                           EKLIP13
 P5=AC(2)*ELL                                                           EKLIP14
 P6=AC(3)*ELL                                                           EKLIP15
 BDE(1)=A(44)*(AC(1)**3)-A(45)*AC(1)                                    EKLIP16
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BDE(2)=A(41)*(AC(1)**4)-A(42)*(AC(1)**2)+A(43)
AM(30)=BDE(1)/BDE(2)
BDE(3)=AM(5)*A(41)+AM(1)*A(42)+A(43)
BDE(4)=AM(6)*A(41)+AM(2)*A(42)
BDE(5)=AM(3)*A(44)+AC(2)*A(45)
BDE(6)=AM(4)*A(44)+AC(3)*A(45)
BDE(7)=BDE(5)*BDE(3)+BDE(6)*BDE(4)
BDE(8)=BDE(6)*BDE(3)-BDE(5)*BDE(4)
BDE(9)=BDE(3)**2+BDE(4)**2
BDE(10)=BDE(7)/BDE(9)
BDE(11)=BDE(8)/BDE(9)
AM(7)=-A(24)*AC(1)+AM(30)*(A(21)*(AC(1)**2)-A(22))
AM(8)=A(23)*(AC(1)**2)
BDE(12)=AM(7)/AM(8)
AM(19)=A(21)*AM(1)+A(22)
AM(20)=A(21)*AM(2)
AM(21)=A(24)*AC(2)
AM(22)=A(24)*AC(3)
BDE(13)=AM(19)*BDE(10)-AM(20)*BDE(11)-AM(21)
BDE(14)=AM(19)*BDE(11)+AM(20)*BDE(10)-AM(22)
BDE(15)=BDE(13)/A(23)
BDE(16)=BDE(14)/A(23)
AM(9)=AM(1)**2+AM(2)**2
AM(10)=AM(1)*BDE(15)+AM(2)*BDE(16)
AM(11)=AM(1)*BDE(16)-AM(2)*BDE(15)
BDE(17)=AM(10)/AM(9)
BDE(18)=AM(11)/AM(9)
AM(13)=-BDE(19)*BDE(12)+BDE(20)*AM(30)-BDE(21)/AC(1)
BDE(27)=BDE(19)*(AM(2)*BDE(18)-AM(1)*BDE(17))+BDE(20)*(AM(1)*BDE(1
10)-AM(2)*BDE(11))-BDE(21)*AC(2)
BDE(28)=-BDE(19)*(AM(2)*BDE(17)+AM(1)*BDE(18))+BDE(20)*(AM(1)*BDE(
111)+AM(2)*BDE(10))-BDE(21)*AC(3)
BDE(29)=AM(1)*BDE(27)+AM(2)*BDE(28)
BDE(30)=AM(1)*BDE(28)-AM(2)*BDE(27)
AM(14)=BDE(29)/AM(9)
AM(15)=BDE(30)/AM(9)
BDE(31)=EXP(P5)*COS(P6)
BDE(32)=EXP(P5)*SIN(P6)
BDE(33)=EXP(-P5)*COS(P6)
BDE(34)=EXP(-P5)*SIN(P6)
BDE(51)=AC(1)*(HA*BDE(12)-HB*AM(30))
BDE(52)=-BDE(18)*HA+BDE(11)*HB
BDE(53)=BDE(17)*HA-BDE(10)*HB
BDE(54)=AC(3)*BDE(52)+AC(2)*BDE(53)
BDE(55)=AC(2)*BDE(52)-AC(3)*BDE(53)
AM(12)=BT(20)/CAPCB-BDE(21)*BT(9)


AM(32)=(CA*NUA+CB*NUB)/R
AM(33)=CA*HA/2.
AM(34)=CA*HB/2.
AM(35)=DA+(H*HA)*CA/4.
AM(36)=DB+(H*HB)*CA/4.
AM(37)=AM(33)+AM(34)
AM(38)=AM(37)*NUA/R
AM(39)=GAMA+GAMB
AM(42)=DA+(CA/4.)*(HA**2)
AM(43)=(HA*HB)*(CA/4.)
BDE(81)=AM(33)*NUA/R
BDE(82)=AM(34)*NUA/R
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      AM(44)=AM(42)-AM(33)*BDE(19)                                      EKLIP77
      AM(45)=AM(43)-AM(33)*BDE(20)                                      EKLIP78
      AM(46)=BDE(81)-AM(33)*BDE(21)                                     EKLIP79
      AM(47)=DB+(CA/4.)*(HB**2)                                         EKLIP80
      AM(48)=AM(43)-AM(34)*BDE(19)                                      EKLIP81
      AM(49)=AM(47)-AM(34)*BDE(20)                                      EKLIP82
      AM(50)=BDE(82)-AM(34)*BDE(21)                                     EKLIP83
      BDE(83)=(H/12.)*(BDE(22)*BDE(19))                                 EKLIP84
      BDE(84)=(H/12.)*(BDE(22)*BDE(20))                                 EKLIP85
      BDE(85)=AM(39)+(H/12.)*(BDE(22)*BDE(21)-BDE(23))                  EKLIP86
                                                                        EKLIP87
      DO 1 I=1,6                                                        EKLIP88
      V(I)=0.0                                                          EKLIP89
      DO 1 J=1,7                                                        EKLIP90
      EN(I,J)=0.0                                                       EKLIP91
    1 CONTINUE                                                          EKLIP92
      EN(1,1)=AC(1)*(AM(44)*BDE(12)-AM(45)*AM(30))+AM(46)               EKLIP93
      EN(1,2)=0.0                                                       EKLIP94
      EN(1,3)=AM(44)*(AC(2)*BDE(17)-AC(3)*BDE(18))+AM(45)*(AC(3)*BDE(11)EKLIP95
     1-AC(2)*BDE(10))+AM(46)                                            EKLIP96
      EN(1,4)=AM(44)*(AC(3)*BDE(17)+AC(2)*BDE(18))-AM(45)*(AC(3)*BDE(10)EKLIP97
     1+AC(2)*BDE(11))                                                   EKLIP98
      EN(1,5)=EN(1,3)                                                   EKLIP99
      EN(1,6)=-EN(1,4)                                                  EKLIP
      EN(2,1)=AC(1)*(BDE(12)*AM(48)-AM(30)*AM(49))+AM(50)               EKLIP
      EN(2,2)=0.0                                                       EKLIP
      EN(2,2)=EN(2,1)                                                   EKLIP
      EN(2,3)=AM(48)*(-AC(3)*BDE(18)+AC(2)*BDE(17))+AM(49)*(AC(3)*BDE(11EKLIP
     1)-AC(2)*BDE(10))+AM(50)                                           EKLIP
      EN(2,4)=AM(48)*(AC(3)*BDE(17)+AC(2)*BDE(18))-AM(49)*(AC(3)*BDE(10)EKLIP
     1+AC(2)*BDE(11))                                                   EKLIP
      EN(2,5)=EN(2,3)                                                   EKLIP
      EN(2,6)=-EN(2,4)                                                  EKLIP
      EN(3,1)=0.0                                                       EKLIP
      EN(3,2)=BDE(85)*AC(1)+BDE(12)*(BDE(83)*AC(1)**2-GAMA)-AM(30)*(BDE(EKLIP
     184)*AC(1)**2-GAMB)                                                EKLIP
      EN(3,3)=BDE(85)*AC(2)-BDE(10)*(GAMB+BDE(84)*AM(1))+BDE(11)*(BDE(84EKLIP
     1)*AM(2))+BDE(17)*(GAMA+AM(1)*BDE(83))-BDE(18)*(BDE(83)*AM(2))     EKLIP
      EN(3,4)=AC(3)*BDE(85)-BDE(11)*(GAMB+BDE(84)*AM(1))-BDE(10)*(BDE(84EKLIP
     1)*AM(2))+BDE(18)*(GAMA+AM(1)*BDE(83))+BDE(17)*(BDE(83)*AM(2))     EKLIP
      EN(3,5)=-EN(3,3)                                                  EKLIP
      EN(3,6)=EN(3,4)                                                   EKLIP
      EN(4,1)=EN(1,1)*COS(P6)                                           EKLIP
      EN(4,2)=EN(1,1)*SIN(P6)                                           EKLIP
      EN(4,3)=EN(1,3)*BDE(31)-EN(1,4)*BDE(32)                           EKLIP
      EN(4,4)=EN(1,3)*BDE(32)+EN(1,4)*BDE(31)                           EKLIP
      EN(4,5)=EN(1,5)*BDE(33)-EN(1,6)*BDE(34)                           EKLIP
      EN(4,6)=EN(1,5)*BDE(34)+EN(1,6)*BDE(33)                           EKLIP
      EN(5,1)=EN(2,1)*COS(P6)                                           EKLIP
      EN(5,2)=EN(2,1)*SIN(P6)                                           EKLIP
      EN(5,3)=EN(2,3)*BDE(31)-EN(2,4)*BDE(32)                           EKLIP
      EN(5,4)=EN(2,3)*BDE(32)+EN(2,4)*BDE(31)                           EKLIP
      EN(5,5)=EN(2,5)*BDE(33)-EN(2,6)*BDE(34)                           EKLIP
      EN(5,6)=EN(2,5)*BDE(34)+EN(2,6)*BDE(33)                           EKLIP
      EN(6,1)=-EN(3,1)*SIN(P6)                                          EKLIP
      EN(6,2)=EN(3,1)*COS(P6)                                           EKLIP
      EN(6,3)=EN(3,3)*BDE(31)-EN(3,4)*BDE(32)                           EKLIP
      EN(6,4)=EN(3,3)*BDE(32)+EN(3,4)*BDE(31)                           EKLIP
      EN(6,5)=EN(3,5)*BDE(33)-EN(3,6)*BDE(34)                           EKLIP
      EN(6,6)=EN(3,5)*BDE(34)+EN(3,6)*BDE(33)                           EKLIP
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                                                                           EKLIP
 EN(1,7)=AM(33)*BT(20)/CAPCB+BDE(81)*BT(9)+BT(18)-(HA/2.)*(BT(5)/((1EKLIP
1.-NUA))-BT(12)/(1.-NUA)+(HA/2.)*BT(21)+AM(46)*BDE(49)                     EKLIP
 EN(2,7)=AM(34)*BT(20)/CAPCB+BDE(82)*BT(9)+BT(19)-(HB/2.)*(BT(5)/((1EKLIP
1.-NUA))-BT(13)/(1.-NUB)+(HB/2.)*BT(22)+AM(50)*BDE(49)                     EKLIP
 EN(1,7)=-EN(1,7)                                                          EKLIP
 EN(2,7)=-EN(2,7)                                                          EKLIP
 EN(3,7)=LOADL                                                             EKLIP
 EN(4,7)=EN(1,7)                                                           EKLIP
 EN(5,7)=EN(2,7)                                                           EKLIP
 EN(6,7)=EN(3,7)                                                           EKLIP
                                                                           EKLIP
 WRITE (9,3) (BDE(I),I=1,100)                                              EKLIP   W
 WRITE (9,5)                                                               EKLIP   W
 WRITE (9,4) (AM(J),J=1,50)                                                EKLIP   W
 WRITE (9,12)                                                              EKLIP   W
 WRITE (9,6) ((EN(I,J),J=1,7),I=1,6)                                       EKLIP   W
 CALL FINGLE (EN,V)                                                        EKLIP
 WRITE (9,7) (I,V(I),I=1,6)                                                EKLIP   W
                                                                           EKLIP
                                                                           EKLIP
 AM(12)=BT(20)/CAPCB-BDE(21)*BDE(49)                                       EKLIP
 PP1=AC(1)*ELL/2.                                                          EKLIP
 PP2=AC(2)*ELL/2.                                                          EKLIP
 PP3=AC(3)*ELL/2.                                                          EKLIP
 CALL CHECK2 (V,AC,PP1,PP2,PP3)                                            EKLIP
 WRITE (9,7) (I,V(I),I=1,6)                                                EKLIP   W
                                                                           EKLIP
 AL(1)=AM(30)*V(1)                                                         EKLIP
 AL(2)=-AM(30)*V(2)                                                        EKLIP
 AL(3)=-BDE(10)*V(3)-BDE(11)*V(4)                                          EKLIP
 AL(4)=BDE(11)*V(3)-BDE(10)*V(4)                                           EKLIP
 AL(5)=BDE(10)*V(5)-BDE(11)*V(6)                                           EKLIP
 AL(6)=BDE(11)*V(5)+BDE(10)*V(6)                                           EKLIP
 AL(7)=-BDE(12)*V(1)                                                       EKLIP
 AL(8)=BDE(12)*V(2)                                                        EKLIP
 AL(7)=BDE(12)*V(1)                                                        EKLIP
 AL(8)=-BDE(12)*V(2)                                                       EKLIP
 AL(9)=BDE(17)*V(3)+BDE(18)*V(4)                                           EKLIP
 AL(10)=-BDE(18)*V(3)+BDE(17)*V(4)                                         EKLIP
 AL(11)=-BDE(17)*V(5)+BDE(18)*V(6)                                         EKLIP
 AL(12)=-BDE(18)*V(5)-BDE(17)*V(6)                                         EKLIP
                                                                           EKLIP
 WRITE (9,8)                                                               EKLIP   W
 WRITE (9,9) (AL(I),I=1,12)                                                EKLIP   W
                                                                           EKLIP
 DO 2 KF=1,11                                                              EKLIP
 X=DELPH(KF)                                                               EKLIP
 IF (X.GT.ELL) GO TO 2                                                     EKLIP
 P1=AC(1)*X                                                                EKLIP
 P2=AC(2)*X                                                                EKLIP
 P3=AC(3)*X                                                                EKLIP
                                                                           EKLIP
                                                                           EKLIP
 CALL DIFF (P1,P2,P3)                                                      EKLIP
                                                                           EKLIP
                                                                           EKLIP
 WW=V(1)*COS(P1)+V(2)*SIN(P1)+EXP(P2)*(V(3)*COS(P3)+V(4)*SIN(P3))+EEKLIP
1XP(-P2)*(V(5)*COS(P3)+V(6)*SIN(P3))+BDE(49)                               EKLIP
 DWW=V(1)*BDE(181)+V(2)*BDE(191)+V(3)*BDE(121)+V(4)*BDE(131)+V(5)*BEKLIP
```

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1DE(141)+V(6)*BDE(151)                                                  EKLIP
 DDWW=V(1)*BDE(182)+V(2)*BDE(192)+V(3)*BDE(122)+V(4)*BDE(132)+V(5)*EKLIP
1BDE(142)+V(6)*BDE(152)                                                 EKLIP
 WB=AL(1)*COS(P1)+AL(2)*SIN(P1)+EXP(P2)*(AL(3)*COS(P3)+AL(4)*SIN(P3EKLIP
1))+EXP(-P2)*(AL(5)*COS(P3)+AL(6)*SIN(P3))                              EKLIP
 DWB=AL(1)*BDE(181)+AL(2)*BDE(191)+AL(3)*BDE(121)+AL(4)*BDE(131)+ALEKLIP
1(5)*BDE(141)+AL(6)*BDE(151)                                            EKLIP
 DDWB=AL(1)*BDE(182)+AL(2)*BDE(192)+AL(3)*BDE(122)+AL(4)*BDE(132)+AEKLIP
1L(5)*BDE(142)+AL(6)*BDE(152)                                           EKLIP
 DDDWB=AL(1)*BDE(183)+AL(2)*BDE(193)+AL(3)*BDE(123)+AL(4)*BDE(133)+EKLIP
1AL(5)*BDE(143)+AL(6)*BDE(153)                                          EKLIP
 WA=AL(7)*COS(P1)+AL(8)*SIN(P1)+EXP(P2)*(AL(9)*COS(P3)+AL(10)*SIN(PEKLIP
13))+EXP(-P2)*(AL(11)*COS(P3)+AL(12)*SIN(P3))                           EKLIP
 DWA=AL(7)*BDE(181)+AL(8)*BDE(191)+AL(9)*BDE(121)+AL(10)*BDE(131)+AEKLIP
1L(11)*BDE(141)+AL(12)*BDE(151)                                         EKLIP
 DDWA=AL(7)*BDE(182)+AL(8)*BDE(192)+AL(9)*BDE(122)+AL(10)*BDE(132)+EKLIP
1AL(11)*BDE(142)+AL(12)*BDE(152)                                        EKLIP
 DDDWA=AL(7)*BDE(183)+AL(8)*BDE(193)+AL(9)*BDE(123)+AL(10)*BDE(133)EKLIP
1+AL(11)*BDE(143)+AL(12)*BDE(153)                                       EKLIP
                                                                        EKLIP
 FNTWW=(1./AC(1))*(V(1)*SIN(P1)-V(2)*COS(P1))+(EXP(P2)/AM(24))*(V(3EKLIP
1)*(AC(2)*COS(P3)+AC(3)*SIN(P3))+V(4)*(AC(2)*SIN(P3)-AC(3)*COS(P3))EKLIP
2)+(EXP(-P2)/AM(24))*(V(5)*(-AC(2)*COS(P3)+AC(3)*SIN(P3))-V(6)*(ACEKLIP
32)*SIN(P3)+AC(3)*COS(P3))))+BDE(49)*X                                  EKLIP
 IF (KF.EQ.1) CALL UCOFF (FNTWW,CONST,0)                                EKLIP
 WU=-BDE(19)*WA-BDE(20)*WB-BDE(21)*FNTWW+(BT(20)/CAPCB)*X+CONST        EKLIP
 DWU=-BDE(19)*DWA-BDE(20)*DWB-BDE(21)*WW+BT(20)/CAPCB                  EKLIP
 DDWU=-BDE(19)*DDWA-BDE(20)*DDWB-BDE(21)*DWW                           EKLIP
 DDDWU=-BDE(19)*DDDWA-BDE(20)*DDDWB-BDE(21)*DDWW                       EKLIP
 TAU=LOAD1-BDE(22)*DDWU-BDE(23)*DWW                                    EKLIP
 DTAU=-BDE(22)*DDDWU-BDE(23)*DDWW                                      EKLIP
 WPJ=LOAD2-BDE(24)*DDDWB-BDE(25)*DTAU+BDE(23)*DWU+BDE(26)*WW           EKLIP
 WRITE (9,10) WB,DWB,DDWB,DDDWB,WA,DWA,DDWA,DDDWA                      EKLIP  W
 WUA=WU+HA*WA/2.+HB*WB/2.                                              EKLIP
 DWUA=DWU+HA*DWA/2.+HB*DWB/2.                                          EKLIP
 DDWUA=DDWU+HA*DDWA/2.+HB*DDWB/2.                                      EKLIP
 WRITE (9,11) WU,DWU,DDWU,WUA,DWUA,DDWUA                               EKLIP  W
 CALL RSLT (X)                                                         EKLIP
 CALL PRINT (X)                                                        EKLIP
2 CONTINUE                                                             EKLIP
 RETURN                                                                EKLIP
                                                                       EKLIP
                                                                       EKLIP
3 FORMAT (1X,9H BDE ARE /(10E12.5))                                    EKLIP
4 FORMAT (10E12.5)                                                     EKLIP
5 FORMAT (20H  AM(J) ARE          )                                    EKLIP
6 FORMAT (1X,7E11.4)                                                   EKLIP
7 FORMAT (4(1X,2HV(I1,2H)=E12.5)/2(1X,2HV(I1,2H)=E12.5))               EKLIP
8 FORMAT (1X,20H AL(I) ARE         )                                   EKLIP
9 FORMAT (6E12.5)                                                      EKLIP
10 FORMAT (/1X,25H BETA(B)DERIVATIVES      /E12.5,1X,E12.5,1X,E12.5,1EKLIP
 1X,E12.5/1X,20H BETA(A) DERIVATIVES/1X,E12.5,1X,E12.5,1X,E12.5,1X,EEKLIP
 212.5//)                                                              EKLIP
11 FORMAT (1X,20H U(B) DERIVATIVES   /1X,E12.5,1X,E12.5,1X,E12.5/1X,2EKLIP
 10HU(A) DERIVATIVES    /1X,E12.5,1X,E12.5,1X,E12.5//)                 EKLIP
12 FORMAT (21H EN(I,J)BY ROWS ARE  )                                   EKLIP
 END                                                                   EKLIP  -
 SUBROUTINE EKLIP4                                                     ***** 1
 DIMENSION JIP(11)                                                     EKLIP 2
 COMMON DLTEMP                                                         EKLIP 3
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      COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,
     1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),BT(30),FNTOA,FN
     2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FM
     3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DW
     4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DW
     5UA
      REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2
      BDE(1)=A(44)*(AC(1)**3)+A(45)*AC(1)
      BDE(2)=A(41)*(AC(1)**4)+A(42)*(AC(1)**2)+A(43)
      BDE(3)=BDE(1)/BDE(2)

      BDE(4)=A(44)*(AC(2)**3)-A(45)*AC(2)
      BDE(5)=A(41)*(AC(2)**4)-A(42)*(AC(2)**2)+A(43)
      BDE(6)=BDE(4)/BDE(5)

      BDE(7)=A(44)*(AC(3)**3)-A(45)*AC(3)
      BDE(8)=A(41)*(AC(3)**4)-A(42)*(AC(3)**2)+A(43)
      BDE(9)=BDE(7)/BDE(8)


      BDE(10)=BDE(3)*(A(21)*(AC(1)**2)+A(22))-A(24)*AC(1)
      BDE(11)=A(23)*(AC(1)**2)
      BDE(12)=BDE(10)/BDE(11)

      BDE(13)=BDE(6)*(A(21)*(AC(2)**2)-A(22))-A(24)*AC(2)
      BDE(14)=A(23)*(AC(2)**2)
      BDE(15)=BDE(13)/BDE(14)

      BDE(16)=BDE(9)*(A(21)*(AC(3)**2)-A(22))-A(24)*AC(3)
      BDE(17)=A(23)*(AC(3)**2)
      BDE(18)=BDE(16)/BDE(17)
      P4=AC(1)*ELL
      P5=AC(2)*ELL
      P6=AC(3)*ELL

      EN(1,1)=(AM(44)*BDE(12)-AM(45)*BDE(3))*AC(1)+AM(46)
      EN(1,2)=EN(1,1)
      EN(1,3)=(AM(44)*BDE(15)-AM(45)*BDE(6))*AC(2)+AM(46)
      EN(1,4)=0.0
      EN(1,5)=(AM(44)*BDE(18)-AM(45)*BDE(9))*AC(3)+AM(46)
      EN(1,6)=0.0
      EN(4,1)=EN(1,1)*EXP(P4)
      EN(4,2)=EN(1,2)*EXP(-P4)
      EN(4,3)=EN(1,3)*COS(P5)
      EN(4,4)=EN(1,3)*SIN(P5)
      EN(4,5)=EN(1,5)*COS(P6)
      EN(4,6)=EN(1,5)*SIN(P6)

      EN(2,1)=(AM(48)*BDE(12)-AM(49)*BDE(3))*AC(1)+AM(50)
      EN(2,2)=EN(2,1)
      EN(2,3)=(AM(48)*BDE(15)-AM(49)*BDE(6))*AC(2)+AM(50)
      EN(2,4)=0.0
      EN(2,5)=(AM(48)*BDE(18)-AM(49)*BDE(9))*AC(3)+AM(50)
      EN(2,6)=0.0
      EN(5,1)=EN(2,1)*EXP(P4)
      EN(5,2)=EN(2,2)*EXP(-P4)
      EN(5,3)=EN(2,3)*COS(P5)
      EN(5,4)=EN(2,3)*SIN(P5)
      EN(5,5)=EN(2,5)*COS(P6)
      EN(5,6)=EN(2,5)*SIN(P6)
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                                                                       EKLIP64
      EN(3,1)=BDE(85)*AC(1)-BDE(3)*(BDE(84)*(AC(1)**2)+GAMB)+BDE(12)*(BDEKLIP65
     1E(83)*(AC(1)**2)+GAMA)                                           EKLIP66
      EN(3,2)=EN(3,1)                                                  EKLIP67
      EN(3,3)=0.0                                                      EKLIP68
      EN(3,4)=BDE(85)*AC(2)-BDE(6)*(BDE(84)*(AC(2)**2)+GAMB)+BDE(15)*(BDEKLIP69
     1E(83)*(AC(2)**2)+GAMA)                                           EKLIP70
      EN(3,5)=0.0                                                      EKLIP71
      EN(3,6)=BDE(85)*AC(3)+BDE(18)*(BDE(83)*(AC(3)**2)-GAMA)-BDE(9)*(BDEKLIP72
     1E(84)*(AC(3)**2)-GAMB)                                           EKLIP73
      EN(6,1)=EN(3,1)*EXP(P4)                                          EKLIP74
      EN(6,2)=EN(3,2)*EXP(-P4)                                         EKLIP75
      EN(6,3)=-EN(3,4)*SIN(P5)                                         EKLIP76
      EN(6,4)=EN(3,4)*COS(P5)                                          EKLIP77
      EN(6,5)=-EN(3,6)*SIN(P6)                                         EKLIP78
      EN(6,6)=EN(3,6)*COS(P6)                                          EKLIP79
                                                                       EKLIP80
      WRITE (9,2) (BDE(I),I=1,20)                                      EKLIP81W
      WRITE (9,3)                                                      EKLIP82W
      WRITE (9,4) ((EN(I,J),J=1,7),I=1,6)                              EKLIP83W
      CALL BINGO (EN,V)                                                EKLIP84
      WRITE (9,5) (I,V(I),I=1,6)                                       EKLIP85W
      AM(12)=BT(20)/CAPCB-BDE(21)*BDE(49)                              EKLIP86
      PP1=AC(1)*ELL/2.                                                 EKLIP87
      PP2=AC(2)*ELL/2.                                                 EKLIP88
      PP3=AC(3)*ELL/2.                                                 EKLIP89
      CALL CHECK4 (V,AC,PP1,PP2,PP3)                                   EKLIP90
      WRITE (9,5) (I,V(I),I=1,6)                                       EKLIP91W
                                                                       EKLIP92
      AL(1)=-BDE(3)*V(1)                                               EKLIP93
      AL(2)=BDE(3)*V(2)                                                EKLIP94
      AL(3)=BDE(6)*V(3)                                                EKLIP95
      AL(4)=-BDE(6)*V(4)                                               EKLIP96
      AL(5)=BDE(9)*V(6)                                                EKLIP97
      AL(6)=-BDE(9)*V(5)                                               EKLIP98
                                                                       EKLIP99
      AL(7)=BDE(12)*V(1)                                               EKLIP
      AL(8)=-BDE(12)*V(2)                                              EKLIP
      AL(9)=-BDE(15)*V(3)                                              EKLIP
      AL(10)=BDE(15)*V(4)                                              EKLIP
      AL(11)=-BDE(18)*V(6)                                             EKLIP
      AL(12)=BDE(18)*V(5)                                              EKLIP
      WRITE (9,6)                                                      EKLIP   W
      WRITE (9,7) (AL(I),I=1,12)                                       EKLIP   W
                                                                       EKLIP
      DO 1 KF=1,11                                                     EKLIP
      X=DELPH(KF)                                                      EKLIP
      P1=AC(1)*X                                                       EKLIP
      P2=AC(2)*X                                                       EKLIP
      P3=AC(3)*X                                                       EKLIP
                                                                       EKLIP
      WW=V(1)*EXP(P1)+V(2)*EXP(-P1)+V(3)*COS(P2)+V(4)*SIN(P2)+V(5)*COS(PEKLIP
     13)+V(6)*SIN(P3)+BDE(49)                                          EKLIP
      DWW=V(1)*BDE(101)+V(2)*BDE(111)+V(3)*BDE(201)+V(4)*BDE(211)+V(5)*BEKLIP
     1DE(221)+V(6)*BDE(231)                                            EKLIP
      DDWW=V(1)*BDE(102)+V(2)*BDE(112)+V(3)*BDE(202)+V(4)*BDE(212)+V(5)*EKLIP
     1BDE(222)+V(6)*BDE(232)                                           EKLIP
      WB=AL(1)*EXP(P1)+AL(2)*EXP(-P1)+AL(3)*COS(P2)+AL(4)*SIN(P2)+AL(5)*EKLIP
     1COS(P3)+AL(6)*SIN(P3)                                            EKLIP
      DWB=AL(1)*BDE(101)+AL(2)*BDE(111)+AL(3)*BDE(201)+AL(4)*BDE(211)+ALEKLIP
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1(5)*BDE(221)+AL(6)*BDF(231)                                             EKLIP
 DDWB=AL(1)*BDE(102)+AL(2)*BDE(112)+AL(3)*BDE(202)+AL(4)*BDE(212)+AEKLIP
1L(5)*BDE(222)+AL(6)*BDE(232)                                            EKLIP
 DDDWB=AL(1)*BDE(103)+AL(2)*BDE(113)+AL(3)*BDE(203)+AL(4)*BDE(213)+EKLIP
1AL(5)*BDE(223)+AL(6)*BDE(233)                                           EKLIP
                                                                         EKLIP
 WA=AL(7)*EXP(P1)+AL(8)*EXP(-P1)+AL(9)*COS(P2)+AL(10)*SIN(P2)+AL(11EKLIP
1)*COS(P3)+AL(12)*SIN(P3)                                                EKLIP
 DWA=AL(7)*BDE(101)+AL(8)*BDE(111)+AL(9)*BDE(201)+AL(10)*BDE(211)+AEKLIP
1L(11)*BDE(221)+AL(12)*BDE(231)                                          EKLIP
 DDWA=AL(7)*BDE(102)+AL(8)*BDE(112)+AL(9)*BDE(202)+AL(10)*BDE(212)+EKLIP
1AL(11)*BDE(222)+AL(12)*BDE(232)                                         EKLIP
 DDDWA=AL(7)*BDE(103)+AL(8)*BDE(113)+AL(9)*BDE(203)+AL(10)*BDE(213)EKLIP
1+AL(11)*BDE(223)+AL(12)*BDE(233)                                        EKLIP
                                                                         EKLIP
 FNTWW=(V(1)*EXP(P1)-V(2)*EXP(-P1))/AC(1)+(V(3)*COS(P2)-V(4)*SIN(P2EKLIP
1))/AC(2)+(V(5)*SIN(P3)-V(6)*COS(P3))/AC(3)+BDE(49)*X                    EKLIP
 IF (KF.EQ.1) CALL UCOFF (FNTWW,CONST,0)                                 EKLIP
 WU=-BDE(19)*WA-BDE(20)*WB-BDE(21)*FNTWW+(BT(20)/CAPCB)*X+CONST          EKLIP
 DWU=-BDE(19)*DWA-BDE(20)*DWB-BDE(21)*WW+BT(20)/CAPCB                    EKLIP
 DDWU=-BDE(19)*DDWA-BDE(20)*DDWB-BDE(21)*DWW                             EKLIP
 DDDWU=-BDE(19)*DDDWA-BDE(20)*DDDWB-BDE(21)*DDWW                         EKLIP
 TAU=LOAD1-BDE(22)*DDWU-BDE(23)*DWW                                      EKLIP
 DTAU=-BDE(22)*DDDWU-BDE(23)*DDWW                                        EKLIP
 WPJ=LOAD2-BDE(24)*DDDWB-BDE(25)*DTAU+BDE(23)*DWU+BDE(26)*WW             EKLIP
 CALL RSLT (X)                                                           EKLIP
 CALL PRINT (X)                                                          EKLIP
 IF (X.EQ.ABS(ELL/2.)) CALL VINSON (X,EA,EB,HA,HB,G3A,G3B,FNXB)          EKLIP
1 CONTINUE                                                               EKLIP
 RETURN                                                                  EKLIP
                                                                         EKLIP
2 FORMAT (1X,9HBDE ARE  /(10E12.5))                                      EKLIP
3 FORMAT (1X,20H EN(I,J) ARE        )                                    EKLIP
4 FORMAT (1X,7E11.4)                                                     EKLIP
5 FORMAT (4(1X,2HV(I1,2H)=E12.5)/2(1X,2HV(I1,2H)=E12.5))                 EKLIP
6 FORMAT (1X,20H AL(I) ARE          )                                    EKLIP
7 FORMAT (6E12.5)                                                        EKLIP
 END                                                                     EKLIP
 SUBROUTINE EKLIPS                                                       ***** 1
 DIMENSION JIP(11)                                                       EKLIP 2
 COMMON DLTEMP                                                           EKLIP 3
 COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,EKLIP 4
1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),BT(30),FNTOA,FNEKLIP 5
2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FMEKLIP 6
3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DWEKLIP 7
4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DWEKLIP 8
5UA                                                                      EKLIP 9
 READ NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2                               EKLIP10
 BDE(1)=A(44)*(AC(1)**3)+A(45)*AC(1)                                     EKLIP11
 BDE(2)=A(41)*(AC(1)**4)+A(42)*(AC(1)**2)+A(43)                          EKLIP12
 BDE(3)=BDE(1)/BDE(2)                                                    EKLIP13
                                                                         EKLIP14
 BDE(4)=A(44)*(AC(2)**3)+A(45)*AC(2)                                     EKLIP15
 BDE(5)=A(41)*(AC(2)**4)+A(42)*(AC(2)**2)+A(43)                          EKLIP16
 BDE(6)=BDE(4)/BDE(5)                                                    EKLIP17
                                                                         EKLIP18
 BDE(7)=A(44)*(AC(3)**3)-A(45)*AC(3)                                     EKLIP19
 BDE(8)=A(41)*(AC(3)**4)-A(42)*(AC(3)**2)+A(43)                          EKLIP20
 BDE(9)=BDE(7)/BDE(8)                                                    EKLIP21
                                                                         EKLIP22
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      BDE(10)=BDE(3)*(A(21)*(AC(1)**2)+A(22))-A(24)*AC(1)
      BDE(11)=A(23)*(AC(1)**2)
      BDE(12)=BDE(10)/BDE(11)

      BDE(13)=BDE(6)*(A(21)*(AC(2)**2)+A(22))-A(24)*AC(2)
      BDE(14)=A(23)*(AC(2)**2)
      BDE(15)=BDE(13)/BDE(14)

      BDE(16)=BDE(9)*(A(21)*(AC(3)**2)-A(22))-A(24)*AC(3)
      BDE(17)=A(23)*(AC(3)**2)
      BDE(18)=BDE(16)/BDE(17)
      P4=AC(1)*ELL
      P5=AC(2)*ELL
      P6=AC(3)*ELL

      EN(1,1)=(AM(44)*BDE(12)-AM(45)*BDE(3))*AC(1)+AM(46)
      EN(1,2)=EN(1,1)
      EN(1,3)=(AM(44)*BDE(15)-AM(45)*BDE(6))*AC(2)+AM(46)
      EN(1,4)=EN(1,3)
      EN(1,5)=(AM(44)*BDE(18)-AM(45)*BDE(9))*AC(3)+AM(46)
      EN(1,6)=0.0
      EN(4,1)=EN(1,1)*EXP(P4)
      EN(4,2)=EN(1,2)*EXP(-P4)
      EN(4,3)=EN(1,3)*EXP(P5)
      EN(4,4)=EN(1,4)*EXP(-P5)
      EN(4,5)=EN(1,5)*COS(P6)
      EN(4,6)=EN(1,5)*SIN(P6)

      EN(2,1)=(AM(48)*BDE(12)-AM(49)*BDE(3))*AC(1)+AM(50)
      EN(2,2)=EN(2,1)
      EN(2,3)=(AM(48)*BDE(15)-AM(49)*BDE(6))*AC(2)+AM(50)
      EN(2,4)=EN(2,3)
      EN(2,5)=(AM(48)*BDE(18)-AM(49)*BDE(9))*AC(3)+AM(50)
      EN(2,6)=0.0
      EN(5,1)=EN(2,1)*EXP(P4)
      EN(5,2)=EN(2,2)*EXP(-P4)
      EN(5,3)=EN(2,3)*EXP(P5)
      EN(5,4)=EN(2,4)*EXP(-P5)
      EN(5,5)=EN(2,5)*COS(P6)
      EN(5,6)=EN(2,5)*SIN(P6)

      EN(3,1)=BDE(85)*AC(1)-BDE(3)*(BDE(84)*(AC(1)**2)+GAMB)+BDE(12)*(BD
     1E(83)*(AC(1)**2)+GAMA)
      EN(3,2)=EN(3,1)
      EN(3,3)=BDE(85)*AC(2)-BDE(6)*(BDE(84)*(AC(2)**2)+GAMB)+BDE(15)*(BD
     1E(83)*(AC(2)**2)+GAMA)
      EN(3,4)=-EN(3,3)
      EN(3,5)=0.0
      EN(3,6)=BDE(85)*AC(3)+BDE(18)*(BDE(83)*(AC(3)**2)-GAMA)-BDE(9)*(BD
     1E(84)*(AC(3)**2)-GAMB)
      EN(6,1)=EN(3,1)*EXP(P4)
      EN(6,2)=EN(3,2)*EXP(-P4)
      EN(6,3)=EN(3,3)*EXP(P5)
      EN(6,4)=EN(3,4)*EXP(-P5)
      EN(6,5)=-EN(3,6)*SIN(P6)
      EN(6,6)=EN(3,6)*COS(P6)

      WRITE (9,2) (BDE(I),I=1,20)
      WRITE (9,3)
```

```
      WRITE (9,4) ((EN(I,J),J=1,7),I=1,6)                              EKLIP83W
      CALL BINGO (EN,V)                                                 EKLIP84
      WRITE (9,5) (I,V(I),I=1,6)                                        EKLIP85W
      AM(12)=BT(20)/CAPCB-BDE(21)*BDE(49)                               EKLIP86
      PP1=AC(1)*ELL/2.                                                  EKLIP87
      PP2=AC(2)*ELL/2.                                                  EKLIP88
      PP3=AC(3)*ELL/2.                                                  EKLIP89
      CALL CHECKS (V,AC,PP1,PP2,PP3)                                    EKLIP90
      WRITE (9,5) (I,V(I),I=1,6)                                        EKLIP91W
                                                                        EKLIP92
      AL(1)=-BDE(3)*V(1)                                                EKLIP93
      AL(2)=BDE(3)*V(2)                                                 EKLIP94
      AL(3)=-BDE(6)*V(3)                                                EKLIP95
      AL(4)=BDE(6)*V(4)                                                 EKLIP96
      AL(5)=BDE(9)*V(6)                                                 EKLIP97
      AL(6)=-BDE(9)*V(5)                                                EKLIP98
                                                                        EKLIP99
      AL(7)=BDE(12)*V(1)                                                EKLIP
      AL(8)=-BDE(12)*V(2)                                               EKLIP
      AL(9)=BDE(15)*V(3)                                                EKLIP
      AL(10)=-BDE(15)*V(4)                                              EKLIP
      AL(11)=-BDE(18)*V(6)                                              EKLIP
      AL(12)=BDE(18)*V(5)                                               EKLIP
      WRITE (9,6)                                                       EKLIP  W
      WRITE (9,7) (AL(I),I=1,12)                                        EKLIP  W
                                                                        EKLIP
      DO 1 KF=1,11                                                      EKLIP
      X=DELPH(KF)                                                       EKLIP
      P1=AC(1)*X                                                        EKLIP
      P2=AC(2)*X                                                        EKLIP
      P3=AC(3)*X                                                        EKLIP
                                                                        EKLIP
      WW=V(1)*EXP(P1)+V(2)*EXP(-P1)+V(3)*EXP(P2)+V(4)*EXP(-P2)+V(5)*COS(EKLIP
     1P3)+V(6)*SIN(P3)+BDE(49)                                          EKLIP
      DWW=V(1)*BDE(101)+V(2)*BDE(111)+V(3)*BDE(161)+V(4)*BDE(171)+V(5)*BEKLIP
     1DE(221)+V(6)*BDE(231)                                             EKLIP
      DDWW=V(1)*BDE(102)+V(2)*BDE(112)+V(3)*BDE(162)+V(4)*BDE(172)+V(5)*EKLIP
     1BDE(222)+V(6)*BDE(232)                                            EKLIP
      WB=AL(1)*EXP(P1)+AL(2)*EXP(-P1)+AL(3)*EXP(P2)+AL(4)*EXP(-P2)+AL(5)EKLIP
     1*COS(P3)+AL(6)*SIN(P3)                                            EKLIP
      DWB=AL(1)*BDE(101)+AL(2)*BDE(111)+AL(3)*BDE(161)+AL(4)*BDE(171)+ALEKLIP
     1(5)*BDE(221)+AL(6)*BDE(231)                                       EKLIP
      DDWB=AL(1)*BDE(102)+AL(2)*BDE(112)+AL(3)*BDE(162)+AL(4)*BDE(172)+AEKLIP
     1L(5)*BDE(222)+AL(6)*BDE(232)                                      EKLIP
      DDDWB=AL(1)*BDE(103)+AL(2)*BDE(113)+AL(3)*BDE(163)+AL(4)*BDE(173)+EKLIP
     1AL(5)*BDE(223)+AL(6)*BDE(233)                                     EKLIP
                                                                        EKLIP
      WA=AL(7)*EXP(P1)+AL(8)*EXP(-P1)+AL(9)*EXP(P2)+AL(10)*EXP(-P2)+AL(1EKLIP
     11)*COS(P3)+AL(12)*SIN(P3)                                         EKLIP
      DWA=AL(7)*BDE(101)+AL(8)*BDE(111)+AL(9)*BDE(161)+AL(10)*BDE(171)+AEKLIP
     1L(11)*BDE(221)+AL(12)*BDE(231)                                    EKLIP
      DDWA=AL(7)*BDE(102)+AL(8)*BDE(112)+AL(9)*BDE(162)+AL(10)*BDE(172)+EKLIP
     1AL(11)*BDE(222)+AL(12)*BDE(232)                                   EKLIP
      DDDWA=AL(7)*BDE(103)+AL(8)*BDE(113)+AL(9)*BDE(163)+AL(10)*BDE(173)EKLIP
     1+AL(11)*BDE(223)+AL(12)*BDE(233)                                  EKLIP
                                                                        EKLIP
      FNTWW=(V(1)*EXP(P1)-V(2)*EXP(-P1))/AC(1)+(V(3)*EXP(P2)-V(4)*EXP(-PEKLIP
     12))/AC(2)+(V(5)*SIN(P3)-V(6)*COS(P3))/AC(3)+BDE(49)*X             EKLIP
      IF (X.EQ.-ELL/2.) CALL UCOFF (FNTWW,CONST)                        EKLIP
      WU=-BDE(19)*WA-BDE(20)*WB-BDE(21)*FNTWW+(BT(20)/CAPCB)*X+CONST    EKLIP
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```
      DWU=-BDE(19)*DWA-BDE(20)*DWB-BDE(21)*WW+BT(20)/CAPCB               EKLIP
      DDWU=-BDE(19)*DDWA-BDE(20)*DDWB-BDE(21)*DWW                        EKLIP
      DDDWU=-BDE(19)*DDDWA-BDE(20)*DDDWB-BDE(21)*DDWW                    EKLIP
      TAU=LOAD1-BDE(22)*DDWU-BDE(23)*DWW                                 EKLIP
      DTAU=-BDE(22)*DDDWU-BDE(23)*DDWW                                   EKLIP
      WPJ=LOAD2-BDE(24)*DDDWB-BDE(25)*DTAU+BDE(23)*DWU+BDE(26)*WW        EKLIP
      CALL RSLT (X)                                                      EKLIP
      CALL PRINT (X)                                                     EKLIP
      QA=(HA/12.)*TAU+GAMA*(WA+DWW)                                      EKLIP
      QB=(HB/12.)*TAU+GAMB*(WB+DWW)                                      EKLIP
      FQ=QA+QB                                                           EKLIP
      WRITE (9,8) QA,QB,FQ                                               EKLIP  W
      IF (X.EQ.ABS(ELL/2.)) CALL VINSON (X,EA,EB,HA,HB,G3A,G3B,FNXB)     EKLIP
    1 CONTINUE                                                           EKLIP
      RETURN                                                             EKLIP
                                                                         EKLIP
    2 FORMAT (1X,9HBDE ARE  /(10E12.5))                                  EKLIP
    3 FORMAT (1X,20H EN(I,J) ARE       . )                               EKLIP
    4 FORMAT (1X,7E11.4)                                                 EKLIP
    5 FORMAT (4(1X,2HV(I1,2H)=E12.5)/2(1X,2HV(I1,2H)=E12.5))             EKLIP
    6 FORMAT (1X,20H AL(I) ARE           )                               EKLIP
    7 FORMAT (6E12.5)                                                    EKLIP
    8 FORMAT (1X,3HQA=E12.5,2X,3HQB=E12.5,2X,5HQBAR=E12.5/)              EKLIP  -
      END                                                                EKLIP
      SUBROUTINE EKLIP6                                                  ***** 1
      DIMENSION JIP(11)                                                  EKLIP 2
      COMMON DLTEMP                                                      EKLIP 3
      COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,EKLIP 4
     1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),C(7),BT(30),FNTCA,FNEKLIP 5
     2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FMEKLIP 6
     3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DWEKLIP 7
     4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DWEKLIP 8
     5UA                                                                EKLIP 9
      REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2                          EKLIP10
C     CASE OF THREE REAL ROOTS,UNEQUAL,POSITIVE                          EKLIP11
      BDE(1)=A(44)*(AC(1)**3)+A(45)*AC(1)                                EKLIP12
      BDE(2)=A(41)*(AC(1)**4)+A(42)*(AC(1)**2)+A(43)                     EKLIP13
      BDE(3)=BDE(1)/BDE(2)                                               EKLIP14
                                                                         EKLIP15
      BDE(4)=A(44)*(AC(2)**3)+A(45)*AC(2)                                EKLIP16
      BDE(5)=A(41)*(AC(2)**4)+A(42)*(AC(2)**2)+A(43)                     EKLIP17
      BDE(6)=BDE(4)/BDE(5)                                               EKLIP18
                                                                         EKLIP19
      BDE(7)=A(44)*(AC(3)**3)+A(45)*AC(3)                                EKLIP20
      BDE(8)=A(41)*(AC(3)**4)+A(42)*(AC(3)**2)+A(43)                     EKLIP21
      BDE(9)=BDE(7)/BDE(8)                                               EKLIP22
                                                                         EKLIP23
      BDE(10)=BDE(3)*(A(21)*(AC(1)**2)+A(22))-A(24)*AC(1)                EKLIP24
      BDE(11)=A(23)*(AC(1)**2)                                           EKLIP25
      BDE(12)=BDE(10)/BDE(11)                                            EKLIP26
                                                                         EKLIP27
      BDE(13)=BDE(6)*(A(21)*(AC(2)**2)+A(22))-A(24)*AC(2)                EKLIP28
      BDE(14)=A(23)*(AC(2)**2)                                           EKLIP29
      BDE(15)=BDE(13)/BDE(14)                                            EKLIP30
                                                                         EKLIP31
      BDE(16)=BDE(9)*(A(21)*(AC(3)**2)+A(22))-A(24)*AC(3)                EKLIP32
      BDE(17)=A(23)*(AC(3)**2)                                           EKLIP33
      BDE(18)=BDE(16)/BDE(17)                                            EKLIP34
                                                                         EKLIP35
                                                                         EKLIP36
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      P4=AC(1)*ELL
      P5=AC(2)*ELL
      P6=AC(3)*ELL

      EN(1,1)=(AM(44)*BDE(12)-AM(45)*BDE(3))*AC(1)+AM(46)
      EN(1,2)=EN(1,1)
      EN(1,3)=(AM(44)*BDE(15)-AM(45)*BDE(6))*AC(2)+AM(46)
      EN(1,4)=EN(1,3)
      EN(1,5)=(AM(44)*BDE(18)-AM(45)*BDE(9))*AC(3)+AM(46)
      EN(1,6)=EN(1,5)

      EN(4,1)=EN(1,1)*EXP(P4)
      EN(4,2)=EN(1,2)*EXP(-P4)
      EN(4,3)=EN(1,3)*EXP(P5)
      EN(4,4)=EN(1,4)*EXP(-P5)
      EN(4,5)=EN(1,5)*EXP(P6)
      EN(4,6)=EN(1,6)*EXP(-P6)

      EN(2,1)=(AM(48)*BDE(12)-AM(49)*BDE(3))*AC(1)+AM(50)
      EN(2,2)=EN(2,1)
      EN(2,3)=(AM(48)*BDE(15)-AM(49)*BDE(6))*AC(2)+AM(50)
      EN(2,4)=EN(2,3)
      EN(2,5)=(AM(48)*BDE(18)-AM(49)*BDE(9))*AC(3)+AM(50)
      EN(2,6)=EN(2,5)

      EN(5,1)=EN(2,1)*EXP(P4)
      EN(5,2)=EN(2,2)*EXP(-P4)
      EN(5,3)=EN(2,3)*EXP(P5)
      EN(5,4)=EN(2,4)*EXP(-P5)
      EN(5,5)=EN(2,5)*EXP(P6)
      EN(5,6)=EN(2,6)*EXP(-P6)

      EN(3,1)=BDE(85)*AC(1)-BDE(3)*(BDE(84)*(AC(1)**2)+GAMB)+BDE(12)*(BD
     1E(83)*(AC(1)**2)+GAMA)
      EN(3,2)=EN(3,1)
      EN(3,3)=BDE(85)*AC(2)-BDE(6)*(BDE(84)*(AC(2)**2)+GAMB)+BDE(15)*(BD
     1E(83)*(AC(2)**2)+GAMA)
      EN(3,4)=-EN(3,3)
      EN(3,5)=BDE(85)*AC(3)-BDE(9)*(BDE(84)*(AC(3)**2)+GAMB)+BDE(18)*(BD
     1E(83)*(AC(3)**2)+GAMA)
      EN(3,6)=-EN(3,5)

      EN(6,1)=EN(3,1)*EXP(P4)
      EN(6,2)=EN(3,2)*EXP(-P4)
      EN(6,3)=EN(3,3)*EXP(P5)
      EN(6,4)=EN(3,4)*EXP(-P5)
      EN(6,5)=EN(3,5)*EXP(P6)
      EN(6,6)=EN(3,6)*EXP(-P6)


      AL(1)=-BDE(3)*V(1)
      AL(2)=BDE(3)*V(2)

      AL(3)=-BDE(6)*V(3)
      AL(4)=BDE(6)*V(4)

      AL(5)=-BDE(9)*V(5)
      AL(6)=BDE(9)*V(6)
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      AL(7)=BDE(12)*V(1)                                                EKLIP97
      AL(8)=-BDE(12)*V(2)                                               EKLIP98
                                                                        EKLIP99
      AL(9)=BDE(15)*V(3)                                                EKLIP
      AL(10)=-BDE(15)*V(4)                                              EKLIP
                                                                        EKLIP
      AL(11)=BDE(18)*V(5)                                               EKLIP
      AL(12)=-BDE(18)*V(6)                                              EKLIP
                                                                        EKLIP
      WW=V(1)*EXP(P1)+V(2)*EXP(-P1)+V(3)*EXP(P2)+V(4)*EXP(-P2)+V(5)*EXP(EKLIP
     1P3)+V(6)*EXP(-P3)+BDE(49)                                         EKLIP
      DWW=V(1)*BDE(101)+V(2)*BDE(111)+V(3)*BDE(161)+V(4)*BDE(171)+V(5)*BEKLIP
     1DE(241)+V(6)*BDE(251)                                             EKLIP
      DDWW=V(1)*BDE(102)+V(2)*BDE(112)+V(3)*BDE(162)+V(4)*BDE(172)+V(5)*EKLIP
     1BDE(242)+V(6)*BDE(252)                                            EKLIP
      WB=AL(1)*EXP(P1)+AL(2)*EXP(-P1)+AL(3)*EXP(P2)+AL(4)*EXP(-P2)+AL(5)EKLIP
     1*EXP(P3)+AL(6)*EXP(-P3)                                           EKLIP
      DWB=AL(1)*BDE(101)+AL(2)*BDE(111)+AL(3)*BDE(161)+AL(4)*BDE(171)+ALEKLIP
     1(5)*BDE(241)+AL(6)*BDE(251)                                       EKLIP
      DDWB=AL(1)*BDE(102)+AL(2)*BDE(112)+AL(3)*BDE(162)+AL(4)*BDE(172)+AEKLIP
     1L(5)*BDE(BDE(242)+AL(6)*BDE(252)                                  EKLIP
      DDDWB=AL(1)*BDE(103)+AL(2)*BDE(113)+AL(3)*BDE(163)+AL(4)*BDE(173)+EKLIP
     1AL(5)*BDE(243)+AL(6)*BDE(253)                                     EKLIP
                                                                        EKLIP
      WA=AL(7)*EXP(P1)+AL(8)*EXP(-P1)+AL(9)*EXP(P2)+AL(10)*EXP(-P2)+AL(1EKLIP
     11)*EXP(P3)+AL(12)*EXP(-P3)                                        EKLIP
      DWA=AL(7)*BDE(101)+AL(8)*BDE(111)+AL(9)*BDE(161.+AL(10)*BDE(171)+AEKLIP
     1L(11)*BDE(241)+AL(12)*BDE(251)                                    EKLIP
      DDWA=AL(7)*BDE(102)+AL(8)*BDE(112)+AL(9)*BDE(162)+AL(10)*BDE(172)+EKLIP
     1AL(11)*BDE(242)+AL(12)*BDE(252)                                   EKLIP
      DDDWA=AL(7)*BDE(103)+AL(8)*BDE(113)+AL(9)*BDE(163)+AL(10)*BDE(173)EKLIP
     1+AL(11)*BDE(243)+AL(12)*BDE(253)                                  EKLIP
      FNTWW=(V(1)*EXP(P1)-V(2)*EXP(-P1))/AC(1)+(V(3)*EXP(P2)-V(4)*EXP(-PEKLIP
     12))/AC(2)+(V(5)*EXP(P3)-V(6)*EXP(-P6))/AC(3)+BDE(49)*X            EKLIP
      IF (X.EQ.-ELL/2.) CALL UCOFF (FNTWW,CONST)                        EKLIP
      WU=-BDE(19)*WA-BDE(20)*WB-BDE(21)*FNTWW+(BT(20)/CAPCB)*X+CONST    EKLIP
      DWU=-BDE(19)*DWA-BDE(20)*DWB-BDE(21)*WW+BT(20)/CAPCB              EKLIP
      DDWU=-BDE(19)*DDWA-BDE(20)*DDWB-BDE(21)*DWW                       EKLIP
      DDDWU=-BDE(19)*DDDWA-BDE(20)*DDDWB-BDE(21)*DDWW                   EKLIP
      TAU=LOAD1-BDE(22)*DDWU-BDE(23)*DWW                                EKLIP
      DTAU=-BDE(22)*DDDWU-BDE(23)*DDWW                                  EKLIP
      WPJ=LOAD2-BDE(24)*DDDWB-BDE(25)*DTAU+BDE(23)*DWU+BDE(26)*WW       EKLIP
                                                                        EKLIP
      CALL RESULT (X)                                                   EKLIP
      CALL PRINT (X)                                                    EKLIP
      RETURN                                                            EKLIP
      END                                                               EKLIP
      SUBROUTINE DIFF (P1,P2,P3)                                        *****

      COMMON DLTEMP
      COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,
     1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),HT(30),FNTOA,FN
     2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FM
     3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DW
     4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DW
     5UA
      REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2
      BDE(101)=AC(1)*EXP(P1)
      BDE(102)=BDE(101)*AC(1)
      BDE(103)=BDE(102)*AC(1)
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```
BDE(104)=BDE(103)*AC(1)


BDE(111)=-AC(1)*EXP(-P1)
BDE(112)=-BDE(111)*AC(1)
BDE(113)=-BDE(112)*AC(1)
BDE(114)=-BDE(113)*AC(1)


BDE(121)=EXP(P2)*(AC(2)*COS(P3)-AC(3)*SIN(P3))
BDE(122)=EXP(P2)*(AM(1)*COS(P3)-AM(2)*SIN(P3))
BDE(123)=EXP(P2)*(AM(3)*COS(P3)-AM(4)*SIN(P3))
BDE(124)=EXP(P2)*(AM(5)*COS(P3)-AM(6)*SIN(P3))


BDE(131)=EXP(P2)*(AC(3)*COS(P3)+AC(2)*SIN(P3))
BDE(132)=EXP(P2)*(AM(2)*COS(P3)+AM(1)*SIN(P3))
BDE(133)=EXP(P2)*(AM(4)*COS(P3)+AM(3)*SIN(P3))
BDE(134)=EXP(P2)*(AM(6)*COS(P3)+AM(5)*SIN(P3))


BDE(141)=-EXP(-P2)*(AC(2)*COS(P3)+AC(3)*SIN(P3))
BDE(142)=+EXP(-P2)*(AM(1)*COS(P3)+AM(2)*SIN(P3))
BDE(143)=-EXP(-P2)*(AM(3)*COS(P3)+AM(4)*SIN(P3))
BDE(144)=+EXP(-P2)*(AM(5)*COS(P3)+AM(6)*SIN(P3))


BDE(151)=+EXP(-P2)*(AC(3)*COS(P3)-AC(2)*SIN(P3))
BDE(152)=-EXP(-P2)*(AM(2)*COS(P3)-AM(1)*SIN(P3))
BDE(153)=+EXP(-P2)*(AM(4)*COS(P3)-AM(3)*SIN(P3))
BDE(154)=+EXP(-P2)*(AM(6)*COS(P3)-AM(5)*SIN(P3))


BDE(161)=AC(2)*EXP(P2)
BDE(162)=AC(2)*BDE(161)
BDE(163)=AC(2)*BDE(162)
BDE(164)=AC(2)*BDE(163)

BDE(171)=-AC(2)*EXP(-P2)
BDE(172)=-AC(2)*BDE(171)
BDE(173)=-AC(2)*BDE(172)
BDE(174)=-AC(2)*BDE(173)


BDE(181)=-AC(1)*SIN(P1)
BDE(182)=-(AC(1)**2)*COS(P1)
BDE(183)=(AC(1)**3)*SIN(P1)
BDE(184)=(AC(1)**4)*COS(P1)


BDE(191)=AC(1)*COS(P1)
BDE(192)=-(AC(1)**2)*SIN(P1)
BDE(193)=-(AC(1)**3)*COS(P1)
BDE(194)=(AC(1)**4)*SIN(P1)


BDE(201)=-AC(2)*SIN(P2)
BDE(202)=-(AC(2)**2)*COS(P2)
BDE(203)=(AC(2)**3)*SIN(P2)
BDE(204)=(AC(2)**4)*COS(P2)
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      BDE(211)=AC(2)*COS(P2)
      BDE(212)=-(AC(2)**2)*SIN(P2)
      BDE(213)=-(AC(2)**3)*COS(P2)
      BDE(214)=(AC(2)**4)*SIN(P2)


      BDE(221)=-AC(3)*SIN(P3)
      BDE(222)=-(AC(3)**2)*COS(P3)
      BDE(223)=(AC(3)**3)*SIN(P3)
      BDE(224)=(AC(3)**4)*COS(P3)


      BDE(231)=AC(3)*COS(P3)
      BDE(232)=-(AC(3)**2)*SIN(P3)
      BDE(233)=-(AC(3)**3)*COS(P3)
      BDE(234)=(AC(3)**4)*SIN(P3)


      BDE(241)=AC(3)*EXP(P3)
      BDE(242)=AC(3)*BDE(241)
      BDE(243)=AC(3)*BDE(242)
      BDE(244)=AC(3)*BDE(243)


      BDE(251)=-AC(3)*EXP(-P3)
      BDE(252)=-AC(3)*BDE(251)
      BDE(253)=-AC(3)*BDE(252)
      BDE(254)=-AC(3)*BDE(253)

      RETURN
      END
      SUBROUTINE POLYR (N,COEFF,ROOTS,D)
      DIMENSION A(51,3), IA(51,3), ROOTS(2,N), D(1), COEFF(1)
      INTEGER DEGREE
      DEGREE=N
      N1=DEGREE+1
      M=10
      MMAX=15
      DELTA=0.0001
      EPSILON=0.000001
      DO 1 I=1,N1
      A(I,1)=COEFF(I)
      IA(I,1)=0
      CALL SCALE (A(I,1),IA(I,1))
    1 CONTINUE
      CALL RSSR (A,IA,ROOTS,DEGREE,M,MMAX,DELTA,EPSILON,D)
      IF (N1-(DEGREE+1)) 3,3,2
    2 RETURN
    3 PRINT 4
      RETURN

    4 FORMAT (21HOSOME ROOTS NOT FOUND)
      END
      SUBROUTINE RSSR (A,IA,ROOTS,DEGREE,M,MMAX,DELTA,EPSILON,D)
      DIMENSION A(51,3), IA(51,3), ROOTS(2,50), D(51), ROMOD(50), MROMOD
     1(50), NONRT(50), MNONRT(50)
      INTEGER DEGREE
      N=DEGREE
```

```
      IF (N) 1,1,3
    1 DEGREE=NCUR
    2 RETURN
    3 N1=N+1
      N2=N1+1
      DO 5 I=1,N
      K=N2-I
      IF (A(K,1)) 6,4,6
    4 J=N1-I
      ROOTS(1,J)=0.0
      ROOTS(2,J)=0.0
    5 CONTINUE
      DEGREE=0
      GO TO 2
    6 N1=K
      N=K-1
      NCUR=N
      NL=N
    7 CALL ROOTSQ (A,IA,NCUR,M)
      CALL REALRT (A,IA,M,NCUR,DELTA,EPSILON,ROMOD,MROMOD,NONRT,MNONRT,N
     1CO,ROOTS)
      IF (NCO) 12,12,8
    8 N1=NCUR+1
      CALL COMPRT (A,IA,ROMOD,ROOTS,M,MNONRT,NONRT,MROMOD,NCO,DELTA,EPSI
     1LON,NCUR)
      IF (NCUR) 12,12,9
    9 IF (NL-NCUR) 11,11,10
   10 NL=NCUR
      GO TO 7
   11 M=M+1
      IF (MMAX-M) 1,7,7
   12 CALL RECON (ROOTS,A(1,1),IA(1,1),D,DEGREE)
      GO TO 1
      END
      SUBROUTINE ROOTSQ (A,IA,NCUR,MM)
      DIMENSION A(51,3), IA(51,3)
      N1=NCUR+1
      DO 1 J=1,N1
      A(J,2)=A(J,1)
      IA(J,2)=IA(J,1)
      A(J,3)=0.0
      IA(J,3)=0
    1 CONTINUE
      DO 10 M=1,MM
      DO 7 J=1,N1
      K1=N1-J
      K2=J-1
      KM=XMINOF(K1,K2)
      IF (KM) 2,5,2
    2 DO 5 L=1,KM
      LR=XMODF(L,2)
      JL=J-L
      JLP=J+L
      IF (LR) 3,3,4
    3 X=A(JL,2)*A(JLP,2)
      IX=IA(JL,2)+IA(JLP,2)
      CALL SCALE (X,IX)
      CALL ADD (A(J,3),IA(J,3),X,IX,A(J,3),IA(J,3))
      GO TO 5
    4 X=A(JL,2)*A(JLP,2)
```

```
      IX=IA(JL,2)+IA(JLP,2)
      CALL SCALE (X,IX)
      CALL SBTRT (A(J,3),IA(J,3),X,IX,A(J,3),IA(J,3))
    5 CONTINUE
      A(J,3)=2.0*A(J,3)
      CALL SCALE (A(J,3),IA(J,3))
      X=A(J,2)**2
      IX=IA(J,2)+IA(J,2)
      CALL SCALE (X,IX)
      CALL ADD (A(J,3),IA(J,3),X,IX,A(J,3),IA(J,3))
      JR=XMODF(J,2)
      IF (JR) 6,6,7
    6 A(J,3)=-A(J,3)
    7 CONTINUE
      IF (MM-M) 10,10,8
    8 DO 9 J=1,N1
      A(J,2)=A(J,3)
      IA(J,2)=IA(J,3)
      A(J,3)=0.0
      IA(J,3)=0
    9 CONTINUE
   10 CONTINUE
      RETURN
      END
      SUBROUTINE REALRT (A,IA,M,NCUR,DELTA,EPSILON,ROMOD,MROMOD,NONRT,MN
     1ONRT,NCO,ROOTS)
      DIMENSION A(51,3), IA(51,3), ROOTS(2,50), ROMOD(50), MROMOD(50), N
     1ONRT(50), MNONRT(50), RATIO(51), IPIV(51), ARED(50), IARED(50)
      RATIO(1)=1.0
      DO 6 I=2,NCUR
      I1=XMODF(I,2)
      IF (A(I,3)) 2,1,2
    1 RATIO(I)=0.0
      GO TO 6
    2 T=A(I,2)*A(I,2)
      IT=IA(I,2)+IA(I,2)
      CALL SCALE (T,IT)
      T=T/A(I,3)
      IT=IT-IA(I,3)
      IF (IT-2) 3,3,1
    3 IF (IT+2) 1,4,4
    4 CALL UNSCALE (T,IT)
      RATIO(I)=T
      IF (I1) 5,5,6
    5 RATIO(I)=-RATIO(I)
    6 CONTINUE
      RATIO(NCUR+1)=1.0
      IPIV(1)=1
      IPIV(NCUR+1)=1
      DO 9 I=2,NCUR
      X=ABSF(RATIO(I)-1.0)
      IF (X-DELTA) 7,8,8
    7 IPIV(I)=1
      GO TO 9
    8 IPIV(I)=0
    9 CONTINUE
      NCUR1=NCUR+1
      I1=0
      MULT=0
      I=1
```

```
      I4=1
   10 I1=I1+1
      I2=I1+I
      MULT=MULT+1
      IF (IPIV(I2)) 10,10,11
   11 ROMOD(I4)=A(I2,3)/A(I,3)
      IROMOD=IA(I2,3)-IA(I,3)
      CALL SCALE (ROMOD(I4),IROMOD)
      IF (ROMOD(I4)) 12,13,13
   12 ROMOD(I4)=-ROMOD(I4)
   13 CALL DOUBLOG (ROMOD(I4),IROMOD,XN,IXN)
      T=2**M*I1
      XN=XN/T
      CALL SCALE (XN,IXN)
      CALL DLEXP (XN,IXN,ROMOD(I4),IROMOD)
      IF (IROMOD-74) 14,14,15
   14 IF (IROMOD+74) 15,15,16
   15 ROMOD(I4)=0.0
      IROMOD=0
      GO TO 17
   16 CALL UNSCALE (ROMOD(I4),IROMOD)
   17 MROMOD(I4)=MULT
      IF (NCUR+1-I2) 19,19,18
   18 I=I2
      I4=I4+1
      MULT=0
      I1=0
      GO TO 10
   19 Q=0.0
      NCO=0
      DO 28 I=1,I4
      KL=I4+1-I
      W=-ROMOD(KL)
      I5=MROMOD(KL)
      DO 26 J=1,I5
      J=J
   20 CALL TEST (A,IA,W,Q,NCUR,ROMOD(KL),EPSILON,K)
      IF (K) 23,23,21
   21 ROOTS(1,NCUR)=-W
      ROOTS(2,NCUR)=0.0
      ARED(1)=A(1,1)
      IARED(1)=IA(1,1)
      DO 22 L=2,NCUR
      Y=ARED(L-1)*W
      IY=IARED(L-1)
      CALL SCALE (Y,IY)
      CALL SBTRT (A(L,1),IA(L,1),Y,IY,ARED(L),IARED(L))
      A(L,1)=ARED(L)
      IA(L,1)=IARED(L)
   22 CONTINUE
      GO TO 25
   23 IF (W) 24,27,27
   24 W=-W
      GO TO 20
   25 NCUR=NCUR-1
   26 CONTINUE
      GO TO 28
   27 NCO=NCO+1
      NONRT(NCO)=KL
      MNONRT(NCO)=I5+1-J
```

```
28 CONTINUE
   RETURN
   END
   SUBROUTINE COMPRT (A,IA,ROMOD,ROOTS,M,MNONRT,NONRT,MROMOD,NCO,DELT
  1A,EPSILON,NCUR)
   DIMENSION A(51,3), IA(51,3), ROMOD(50), ROOTS(2,50), SR(51,3), ISR
  1(51,3), SROMOD(50), SROOTS(2,50), MNONRT(50), NONRT(50), MSROMOD(5
  20), NSONRT(49), MSNORT(49), MROMOD(50), U(2), R(2), B(49)
   DO 28 I=1,NCO
   JA=NONRT(I)
   I1=MNONRT(I)
   I1=I1/2
   IF (I1) 1,1,2
 1 I1=1
 2 IF (ROMOD(JA)) 3,28,3
 3 Q=ROMOD(JA)
   DO 27 J=1,I1
   CALL SUBRES (A,IA,NCUR,SR,ISR,Q)
   IF (NCUR-4) 5,4,6
 4 NSCUR=2
   GO TO 7
 5 NSCUR=1
   J2=1
   GO TO 8
 6 NSCUR=NCUR-3
 7 J2=NSCUR
 8 LL=NSCUR+1
   IF (NSCUR-1) 9,9,11
 9 IF (SR(1,1)) 10,12,10
10 X=SR(1,1)
   IX=ISR(1,1)
   Y=SR(2,1)
   IY=ISR(2,1)
   CALL UNSCALE (X,IX)
   CALL UNSCALE (Y,IY)
   SROOTS(1,1)=-Y/X
   NSCUR=0
   GO TO 13
11 CALL ROOTSO (SR,ISR,NSCUR,M)
   CALL REALRT (SR,ISR,M,NSCUR,DELTA,EPSILON,SROMOD,MSROMOD,NSONRT,MS
  1NORT,NSCO,SROOTS)
   IF (J2-NSCUR) 12,12,13
12 SROOTS(1,J2)=0.0
13 SROOTS(1,J2)=SROOTS(1,J2)*ROMOD(JA)
   T=ROMOD(JA)+ROMOD(JA)
   IF (SROOTS(1,J2)-T) 14,21,21
14 W=SROOTS(1,J2)
   WE=ROMOD(JA)*ROMOD(JA)
   CALL TEST (A,IA,W,WE,NCUR,ROMOD(JA),EPSILON,K)
   IF (K) 20,20,15
15 ROOTS(1,NCUR)=-W/2.0
   T=4.0*WE
   U=W*W
   T=T-U
   IF (T) 16,16,17
16 T=-T
   U=SQRT(T)
   ROOTS(1,NCUR)=ROOTS(1,NCUR)-U/2.0
   ROOTS(1,NCUR-1)=-(W-U)/2.0
   ROOTS(2,NCUR)=0.0
```

```
      ROOTS(2,NCUR-1)=0.0
      GO TO 18
   17 U=SQRT(T)
      ROOTS(2,NCUR)=U/2.0
      ROOTS(1,NCUR-1)=ROOTS(1,NCUR)
      ROOTS(2,NCUR-1)=-ROOTS(2,NCUR)
   18 D(1)=W
      D(2)=WE
      CALL QUADIV (NCUR,A,IA,R,D,B)
      JX=NCUR-1
      DO 19 JY=1,JX
      A(JY,1)=B(JY)
      IA(JY,1)=0
      CALL SCALE (A(JY,1),IA(JY,1))
   19 CONTINUE
      NCUR=NCUR-2
      GO TO 27
   20 W=-W
      CALL TEST (A,IA,W,WE,NCUR,ROMOD(JA),EPSILON,K)
      IF (K) 21,21,15
   21 IF (J2-(NSCUR+1)) 28,22,24
   22 IF (J2-1) 26,28,23
   23 J2=J2-1
      SROOTS(1,J2)=0.0
      GO TO 14
   24 IF (SROOTS(1,J2)-SROOTS(1,J2-1)) 25,26,25
   25 J2=J2-1
      GO TO 13
   26 J2=J2-1
      GO TO 21
   27 CONTINUE
   28 CONTINUE
      RETURN
      END
      SUBROUTINE TEST (A,IA,W,Q,N,ROMOD,EPSILON,K)
      DIMENSION A(51,3), IA(51,3), B(3), IB(3), T(2), E(2), C(51)
      B(1)=0.0
      IX=0
      IW=0
      IB(1)=0
      B(2)=A(1,1)
      IB(2)=IA(1,1)
      DO 2 I=1,N
      X=W*B(2)
      IX=IB(2)
      CALL SCALE (X,IX)
      Y=Q*B(1)
      IY=IB(1)
      CALL SCALE (Y,IY)
      CALL ADD (X,IX,Y,IY,Z,IZ)
      CALL SBTRT (A(I+1,1),IA(I+1,1),Z,IZ,B(3),IB(3))
      IF (N-I) 2,2,1
    1 B(1)=B(2)
      IB(1)=IB(2)
      B(2)=B(3)
      IB(2)=IB(3)
    2 CONTINUE
      KOUNT=1
      CEPSIL=EPSILON
      T(1)=0.0$T(2)=0.0
```

```
      N1=N+1
      X=2.0*EPSILON
      Y=X*ROMOD
      E(1)=ROMOD+Y
      E(2)=ROMOD+CEPSIL*ROMOD
      DO 6 I=1,N1
      IF (A(I,1)) 3,4,4
    3 C(I)=-A(I,1)$IC=IA(I,1)
      GO TO 5
    4 C(I)=A(I,1)$IC=IA(I,1)
    5 CALL UNSCALE (C(I),IC)
      T(1)=T(1)*E(1)+C(I)
      T(2)=T(2)*E(2)+C(I)
    6 CONTINUE
      DIF=T(1)-T(2)
      IF (Q) 18,7,18
    7 IF (B(3)) 8,9,9
    8 B(3)=-B(3)
    9 IF (IB(3)-74) 10,10,12
   10 IF (IB(3)+74) 12,11,11
   11 CALL UNSCALE (B(3),IB(3))
      IF (DIF-B(3)) 12,12,17
   12 K=0
      IF (KOUNT-2) 13,16,16
   13 IF (Q) 15,14,15
   14 IF (W) 15,16,16
   15 SENSE LIGHT 2
      KOUNT=KOUNT+1
   16 RETURN
   17 K=1
      GO TO 16
   18 IF (IB(2)-74) 19,19,12
   19 IF (IB(2)+74) 12,20,20
   20 CALL UNSCALE (B(2),IB(2))
      IF (IB(3)-74) 21,21,12
   21 IF (IB(3)+74) 12,22,22
   22 CALL UNSCALE (B(3),IB(3))
      X=Q*B(2)*B(2)
      Y=W*B(2)*B(3)
      Z=B(1)*B(3)
      Y=X-Y+Z
      IF (Y) 23,17,24
   23 Y=-Y
   24 DIF=DIF*DIF
      IF (DIF-Y) 12,17,17
      END
      SUBROUTINE SUBRES (A,IA,N,SR,ISR,ROMOD)
      DIMENSION A(51,3), IA(51,3), SR(51,3), ISR(51,3), C(51), B(50,3)
      N1=N+1
      T=1.0
      DO 1 I=1,N
      J=N1-I
      T=T*ROMOD
      C(J)=A(J,1)*T
      IC=IA(J,1)
      CALL UNSCALE (C(J),IC)
    1 CONTINUE
      C(N1)=A(N1,1)
      IC=IA(N1,1)
      CALL UNSCALE (C(N1),IC)
```

```
      IF (N-2) 17,17,2
    2 N2=N-2
      DO 3 I=1,N2
      B(I,1)=0.0
      B(I,2)=0.0
    3 CONTINUE
      I=2
      B(1,2)=C(1)
    4 B(1,3)=C(I)-B(1,1)
      DO 5 J=2,N2
      B(J,3)=-B(J-1,2)-B(J,1)
    5 CONTINUE
      IF (N-(3+I)) 8,6,6
    6 I=I+1
      DO 7 J=1,N2
      B(J,1)=B(J,2)
      B(J,2)=B(J,3)
    7 CONTINUE
      GO TO 4
    8 IF (N-4) 19,9,13
    9 IF (N-(2+I)) 12,10,10
   10 I=I+1
      DO 11 J=1,2
      B(J,1)=B(J,2)
      B(J,2)=B(J,3)
   11 CONTINUE
      GO TO 4
   12 B(3,3)=-B(2,2)
      SR(3,1)=-C(5)+B(1,3)
      ISR(3,1)=0
      SR(2,1)=B(2,3)
      ISR(2,1)=0
      SR(1,1)=B(3,3)
      ISR(1,1)=0
      CALL SCALE (SR(1,1),ISR(1,1))
      CALL SCALE (SR(2,1),ISR(2,1))
      CALL SCALE (SR(3,1),ISR(3,1))
      GO TO 16
   13 SR(N2,1)=C(N)-B(1,3)
      ISR(N2,1)=0
      SR(N2-1,1)=-C(N1)-B(2,3)
      ISR(N2-1,1)=0
      CALL SCALE (SR(N2,1),ISR(N2,1))
      CALL SCALE (SR(N2-1,1),ISR(N2-1,1))
      IF (N2-2) 16,16,14
   14 DO 15 J=3,N2
      K=N2+1-J
      SR(K,1)=-B(J,3)
      ISR(K,1)=0
      CALL SCALE (SR(K,1),ISR(K,1))
   15 CONTINUE
   16 RETURN
   17 SR(1,1)=C(1)
      ISR(1,1)=0
      SR(2,1)=-C(2)
      ISR(2,1)=0
   18 CALL SCALE (SR(1,1),ISR(1,1))
      CALL SCALE (SR(2,1),ISR(2,1))
      GO TO 16
   19 SR(1,1)=-C(4)
```

```
      SR(2,1)=C(3)-C(1)
      ISR(1,1)=0
      ISR(2,1)=0
      GO TO 18
      END
      SUBROUTINE RECON (ROOTS,A,IA,D,N)
      DIMENSION ROOTS(2,50), D(51)
      X=A
      IX=IA
      CALL UNSCALE (X,IX)
      DO 1 I=1,N
      D(I)=0.0
    1 CONTINUE
      D(N+1)=1.0
      I=1
      NL=N-1
    2 IF (ROOTS(2,I)) 3,7,3
    3 T=ROOTS(1,I)*ROOTS(1,I)
      U=ROOTS(2,I)*ROOTS(2,I)
      T=T+U
      U=2.0*ROOTS(1,I)
      DO 5 J=1,NL
      IF (I+J-N) 5,4,4
    4 D(J)=D(J+2)+T*D(J)
      D(J)=D(J)-U*D(J+1)
    5 CONTINUE
      D(N)=T*D(N)
      D(N)=D(N)-U*D(N+1)
      D(N+1)=T*D(N+1)
      I=I+2
    6 IF (N-I) 10,2,2
    7 DO 9 J=1,N
      IF (J+I-N) 9,9,8
    8 D(J)=D(J+1)-D(J)*ROOTS(1,I)
    9 CONTINUE
      D(N+1)=-D(N+1)*ROOTS(1,I)
      I=I+1
      GO TO 6
   10 NS=N+1
      DO 11 II=1,NS
      D(II)=D(II)*X
   11 CONTINUE
      RETURN
      END
      SUBROUTINE QUADIV (N,A,IA,R,D,B)
      DIMENSION A(51,3), IA(51,3), R(2), D(2), B(49)
      B(1)=A(1,1)
      IB=IA(1,1)
      CALL UNSCALE (B(1),IB)
      IF (N-2) 4,4,1
    1 AA=A(2,1)
      IAA=IA(2,1)
      CALL UNSCALE (AA,IAA)
      B(2)=AA-R(1)*D(1)
      IF (N-3) 4,4,2
    2 NT=N-1
      DO 3 I=3,NT
      XN=R(I-1)*D(1)
      YN=B(I-2)*D(2)
      AA=A(I,1)
```

```
      IAA=IA(I,1)
      CALL UNSCALE (AA,IAA)
      B(I)=AA-(XN+YN)
    3 CONTINUE
    4 XN=B(N-1)*D(1)
      YN=B(N-2)*D(2)
      AA=A(N,1)
      IAA=IA(N,1)
      CALL UNSCALE (AA,IAA)
      R(1)=AA-(XN+YN)
      AA=A(N+1,1)
      IAA=IA(N+1,1)
      CALL UNSCALE (AA,IAA)
      R(2)=AA-B(N-1)*D(2)
      RETURN
      END
      SUBROUTINE DOUBLOG (X,IX,Y,IY)
      T=64.0
      IF (X) 1,2,3
    1 PRINT 5
    2 Y=0.0
      IY=0
      GO TO 4
    3 TO=IX
      Y=ALOG(X)+TO*ALOG(T)
      IY=0
      CALL SCALE (Y,IY)
    4 RETURN

    5 FORMAT (46H0THE LOG OF A NON-POSITIVE NUMBER IS REQUESTED)
      END
      SUBROUTINE DLEXP (X,IX,Z,IZ)
      Z=EXP(X)
      IZ=0
      IF (IX) 1,8,6
    1 I=-IX
      II=6*I
      DO 4 J=1,II
      K=XMODF(IZ,2)
      IF (K) 2,3,2
    2 IZ=IZ-1
      Z=64.0*Z
    3 IZ=IZ/2
      Z=SQRT(Z)
      CALL SCALE (Z,IZ)
    4 CONTINUE
    5 RETURN
    6 I=6*IX
      DO 7 J=1,I
      Z=Z*Z
      IZ=IZ+IZ
      CALL SCALE (Z,IZ)
    7 CONTINUE
      GO TO 5
    8 CALL SCALE (Z,IZ)
      GO TO 5
      END
      SUBROUTINE ADD (X,IX,Y,IY,Z,IZ)
      IF (X) 3,1,3
    1 Z=Y
```

```
      IZ=IY
    2 RETURN
    3 IF (Y) 5,4,5
    4 Z=X
      IZ=IX
      GO TO 2
    5 IDIFF=IX-IY
      IF (IDIFF) 6,7,7
    6 IA=IY
      A=Y
      B=X
      IDIFF=-IDIFF
      GO TO 8
    7 IA=IX
      A=X
      B=Y
    8 IF (16-IDIFF) 9,9,10
    9 Z=A
      IZ=IA
      GO TO 2
   10 IF (IDIFF) 11,13,11
   11 DO 12 I=1,IDIFF
      B=B/64.0
   12 CONTINUE
   13 CONTINUE
      Z=A+B
      IZ=IA
      CALL SCALE (Z,IZ)
      GO TO 2
      END
      SUBROUTINE SBTRT (X,IX,Y,IY,Z,IZ)
      W=-Y
      CALL ADD (X,IX,W,IY,Z,IZ)
      RETURN
      END
      SUBROUTINE SCALE (X,IX)
      REC64=1.0/64.0
      IF (X) 1,11,2
    1 Y=-X
      GO TO 3
    2 Y=X
    3 IF (64.0-Y) 4,5,5
    4 Y=Y/64.0
      IX=IX+1
      GO TO 3
    5 IF (Y-REC64) 6,7,7
    6 Y=Y*64.0
      IX=IX-1
      GO TO 5
    7 IF (X) 8,9,9
    8 X=-Y
      GO TO 10
    9 X=Y
   10 RETURN
   11 IX=0
      GO TO 10
      END
      SUBROUTINE UNSCALE (X,IX)
      IF (IX+64) 1,2,2
    1 X=0.0
```

```
      IX=0
      GO TO 6
    2 IF (IX-84) 4,4,3
    3 X=1.0E+153
      IX=0
      PRINT 7
      GO TO 6
    4 IF (IX) 5,6,5
    5 X=X*64.0**IX
      IX=0
    6 RETURN

    7 FORMAT (25HOEXP. OVERFLOW IN UNSCALE)
      END
      SUBROUTINE CHECK1 (V,AC,PP1,PP2,PP3)
      DIMENSION V(6), AC(3)
      V(1)=V(1)*EXP(PP1)
      V(2)=V(2)*EXP(-PP1)
      AV3=EXP(PP2)*(V(3)*COS(PP3)+V(4)*SIN(PP3))
      AV4=EXP(PP2)*(V(4)*COS(PP3)-V(3)*SIN(PP3))
      AV5=EXP(-PP2)*(V(5)*COS(PP3)+V(6)*SIN(PP3))
      AV6=EXP(-PP2)*(V(6)*COS(PP3)-V(5)*SIN(PP3))
      V(3)=AV3
      V(4)=AV4
      V(5)=AV5
      V(6)=AV6

      RETURN
      END
      SUBROUTINE CHECK2 (V,AC,PP1,PP2,PP3)
      DIMENSION V(6), AC(3)
      AV1=V(1)*COS(PP1)+V(2)*SIN(PP1)
      AV2=-V(1)*SIN(PP1)+V(2)*COS(PP1)
      V(1)=AV1
      V(2)=AV2
      AV3=EXP(PP2)*(V(3)*COS(PP3)+V(4)*SIN(PP3))
      AV4=EXP(PP2)*(V(4)*COS(PP3)-V(3)*SIN(PP3))
      AV5=EXP(-PP2)*(V(5)*COS(PP3)+V(6)*SIN(PP3))
      AV6=EXP(-PP2)*(V(6)*COS(PP3)-V(5)*SIN(PP3))
      V(3)=AV3
      V(4)=AV4
      V(5)=AV5
      V(6)=AV6
      RETURN
      END
      SUBROUTINE CHECK4 (V,AC,PP1,PP2,PP3)
      DIMENSION V(6), AC(3)
      V(1)=V(1)*EXP(PP1)
      V(2)=V(2)*EXP(-PP1)
      AV3=V(3)*COS(PP2)+V(4)*SIN(PP2)
      AV4=-V(3)*SIN(PP2)+V(4)*COS(PP2)
      AV5=V(5)*COS(PP3)+V(6)*SIN(PP3)
      AV6=-V(5)*SIN(PP3)+V(6)*COS(PP3)
      V(3)=AV3
      V(4)=AV4
      V(5)=AV5
      V(6)=AV6
      RETURN
      END
      SUBROUTINE CHECK5 (V,AC,PP1,PP2,PP3)
```

```
      DIMENSION V(6), AC(3)
      V(1)=V(1)*EXP(PP1)
      V(2)=V(2)*EXP(-PP1)
      V(3)=V(3)*EXP(PP2)
      V(4)=V(4)*EXP(-PP2)
      AV5=V(5)*COS(PP3)+V(6)*SIN(PP3)
      AV6=-V(5)*SIN(PP3)+V(6)*COS(PP3)
      V(5)=AV5
      V(6)=AV6
      RETURN
      END
      SUBROUTINE RICF (ROOT,AC,KLIP)
      DIMENSION ICO(6), IIM(6), IRE(6)
      DIMENSION ROOT(2,6), AC(3)
      COMMON /KLP/ IC1,IC2,IC3,IC4,IC5,IC6,IM1,IM2,IM3,IM4,IM5,IM6,IR1,I
     1R2,IR3,IR4,IR5,IR6
      EQUIVALENCE (IC1,ICO(1)), (IM1,IIM(1)), (IR1,IRE(1))
      WRITE (9,20)
      WRITE (9,21) ((ROOT(I,J),I=1,2),J=1,6)
      DO 1 J=1,6
      ICO(J)=0
      IIM(J)=0
      IRE(J)=0
    1 CONTINUE
      IR=0
      IC=0
      II=0
      DO 5 J=1,6
      IF (ABS(ROOT(1,J)).LT.1.E-12) ROOT(1,J)=0.0
      IF (ABS(ROOT(2,J)).LT.1.E-12) ROOT(2,J)=0.0
      IF (ROOT(2,J).EQ.0.0) GO TO 4
      IF (ROOT(1,J)) 2,3,2
    2 IC=IC+1
      ICO(IC)=J
      GO TO 5
    3 II=II+1
      IIM(II)=J
      GO TO 5
    4 IR=IR+1
      IRE(IR)=J
    5 CONTINUE
      WRITE (9,22) IR,II,IC
      IF (IR.EQ.2.AND.IC.EQ.4) GO TO 6
      IF (IR.EQ.2.AND.IM.EQ.4) GO TO 7
      IF (IM.EQ.6) GO TO 9
      IF (IP.EQ.2.AND.IC.EQ.4) GO TO 16
      IF (IR.EQ.4.AND.IM.EQ.2) GO TO 17
      IF (IR.EQ.6) GO TO 19
      GO TO 19


    6 KLIP=1

      AC(1)=ABS(ROOT(1,IR1))
      AC(2)=ABS(ROOT(1,IC1))
      AC(3)=ABS(ROOT(2,IC1))
      RETURN
    7 KLIP=2

      AC(1)=ABS(ROOT(1,IR1))
```

```
      AC(2)=ABS(ROOT(2,IM1))
      IF (AC(2).NE.ABS(ROOT(2,IM2))) GO TO 8
      AC(3)=ABS(ROOT(2,IM3))
      RETURN
    8 AC(3)=ABS(ROOT(2,IM2))
      RETURN

    9 KLIP=3

      AC(1)=ABS(ROOT(2,IM1))
      IF (ABS(ROOT(2,IM1)).NE.ABS(ROOT(2,IM2))) GO TO 10
      AC(2)=ABS(ROOT(2,IM3))
      IF (ABS(ROOT(2,IM4)).NE.AC(1).AND.ABS(ROOT(2,IM4)).NE.AC(2)) GO TO
     1R12
      GO TO 14
   10 AC(2)=ABS(ROOT(2,IM2))
      IF (ABS(ROOT(2,IM3)).EQ.AC(1)) GO TO 11
      IF (ABS(ROOT(2,IM3)).EQ.AC(2)) GO TO 13
      AC(3)=ABS(ROOT(2,IM3))
      RETURN
   11 IF (ABS(ROOT(2,IM4)).EQ.AC(2)) GO TO 14
   12 AC(3)=ABS(ROOT(2,IM4))
      RETURN
   13 IF (ABS(ROOT(2,IM4)).NE.AC(1)) GO TO 15
   14 AC(3)=ABS(ROOT(2,IM5))
      RETURN
   15 AC(3)=ABS(ROOT(2,IM4))
      RETURN

   16 KLIP=4

      AC(1)=ABS(ROOT(2,IM1))
      AC(2)=ABS(ROOT(1,IC1))
      AC(3)=ABS(ROOT(2,IC1))
      RETURN

   17 KLIP=5

      AC(1)=ABS(ROOT(1,IR1))
      AC(3)=ABS(ROOT(2,IM2))
      IF (ABS(ROOT(1,IR2)).EQ.AC(1)) GO TO 18
      AC(2)=ABS(ROOT(1,IR2))
      RETURN
   18 AC(2)=ABS(ROOT(1,IR3))
   19 KLIP=1776
      RETURN

   20 FORMAT (10X,10H ROOTS     )
   21 FORMAT (1X,E12.5,12X,E12.5)
   22 FORMAT (10X,3HIR=I4,3X,3HII=I4,3X,3HIC=I4/)
      END
      SUBROUTINE FINGLE (EN,V)

      DIMENSION FEN(3,3), VP(3)
      DIMENSION DEN(3,3), EN(6,7), V(6), DUDU(6), C(3,1)
      DO 1 I=1,3
      DO 1 J=1,3
      DEN(I,J)=0.0
    1 CONTINUE
      DO 2 I=1,3
```

```
      DEN(I,1)=EN(I+3,1)
      DEN(I,2)=EN(I+3,3)
      DEN(I,3)=EN(I+3,4)
      C(I,1)=EN(I+3,7)
    2 CONTINUE
      WRITE (9,9) ((DEN(I,J),J=1,3),I=1,3)
      CALL LEQ (DEN,C,3,1,3,3,DET)
      V(1)=C(1,1)
      V(3)=C(2,1)
      V(4)=C(3,1)
      WRITE (9,10) V(1),V(3),V(4)
      DO 3 I=1,3
      DO 3 J=1,3
      FEN(I,J)=0.0
    3 CONTINUE
      DO 4 I=1,3
      FEN(I,1)=EN(I,2)
      FEN(I,2)=EN(I,5)
      FEN(I,3)=EN(I,6)
    4 CONTINUE
      C(1,1)=EN(1,7)-EN(1,1)*V(1)-EN(1,3)*V(3)-EN(1,4)*V(4)
      C(2,1)=EN(2,7)-EN(2,1)*V(1)-EN(2,3)*V(3)-EN(2,4)*V(4)
      C(3,1)=EN(3,7)-EN(3,1)*V(1)-EN(3,3)*V(3)-EN(3,4)*V(4)
      WRITE (9,8) ((FEN(I,J),J=1,3),I=1,3)
      WRITE (9,6) C(1,1),C(2,1),C(3,1)
      CALL LEQ (FEN,C,3,1,3,3,DET)
      V(2)=C(1,1)
      V(5)=C(2,1)
      V(6)=C(3,1)
      WRITE (9,11) V(2),V(5),V(6)
      DO 5 I=1,6
      DUDU(I)=EN(I,1)*V(1)+EN(I,2)*V(2)+EN(I,3)*V(3)+EN(I,4)*V(4)+EN(I,5
     1)*V(5)+EN(I,6)*V(6)-EN(I,7)
    5 CONTINUE
      WRITE (9,7) (DUDU(I),I=1,6)
      RETURN

    6 FORMAT (1X,25H FOR THE FEN MATRIX       /10X,7HC(1,1)=E12.5,5X,7HC(
     12,1)=E12.5,5X,7HC(3,1)=E12.5/)
    7 FORMAT (1X,36H SOLUTION CHECK BY SUBSTITUTION  IS /(6E12.5))
    8 FORMAT (1X,20H FEN(I,J) BY ROWS IS/(3E15.7))
    9 FORMAT (1X,20H DEN(I,J) BY ROWS IS/(3E15.7))
   10 FORMAT (1X,5HV(1)=E12.5,2X,5HV(3)=E12.5,2X,5HV(4)=E12.5)
   11 FORMAT (1X,5HV(2)=E12.5,2X,5HV(5)=E12.5,2X,5HV(6)=E12.5)
      END
      SUBROUTINE LEQ (A,B,NEQS,NSOLNS,IA,IB,DET)                    *****
      LINEAR EQUATIONS SOLUTIONS  FORTRAN II VERSION
      SOLVE A SYSTEM OF LINEAR EQUATIONS OF THE FORM AX=B BY A MODIFIED
      GAUSS ELIMINATION SCHEME

      NEQS = NUMBER OF EQUATIONS AND UNKNOWNS
      NSOLNS = NUMBER OF VECTOR SOLUTIONS DESIRED
      IA = NUMBER OF ROWS OF A AS DEFINED BY DIMENSION STATEMENT ENTRY
      IB = NUMBER OF ROWS OF B AS DEFINED BY DIMENSION STATEMENT ENTRY
      ADET = DETERMINANT OF A, AFTER EXIT FROM LEQ

      DIMENSION A(IA,IA), B(IB,IB)
      NSIZ=NEQS
      NBSIZ=NSOLNS
      NORMALIZE EACH ROW BY ITS LARGEST ELEMENT. FORM PARTIAL DETERNT
```

```
      DET=1.0
      DO 6 I=1,NSIZ
      BIG=A(I,1)
      IF (NSIZ-1) 17,17,1
    1 DO 3 J=2,NSIZ
      IF (ABSF(BIG)-ABSF(A(I,J))) 2,3,3
    2 BIG=A(I,J)
    3 CONTINUE
      BG=1.0/BIG
      DO 4 J=1,NSIZ
    4 A(I,J)=A(I,J)*BG
      DO 5 J=1,NBSIZ
    5 B(I,J)=B(I,J)*BG
      DET=DET*BIG
    6 CONTINUE
      START SYSTEM REDUCTION
      NUMSYS=NSIZ-1
      DO 16 I=1,NUMSYS
      SCAN FIRST COLUMN OF CURRENT SYSTEM FOR LARGEST ELEMENT
      CALL THE ROW CONTAINING THIS ELEMENT, ROW NBGRW
      NN=I+1
      BIG=A(I,I)
      NBGRW=I
      DO 8 J=NN,NSIZ
      IF (ABSF(BIG)-ABSF(A(J,I))) 7,8,8
    7 BIG=A(J,I)
      NBGRW=J
    8 CONTINUE
      BG=1.0/BIG
      SWAP ROW I WITH ROW NBGRW UNLESS I=NBGRW
      IF (NBGRW-I) 9,12,9
      SWAP A-MATRIX ROWS
    9 DO 10 J=I,NSIZ
      TEMP=A(NBGRW,J)
      A(NBGRW,J)=A(I,J)
   10 A(I,J)=TEMP
      DET=-DET
      SWAP B-MATRIX ROWS
      DO 11 J=1,NBSIZ
      TEMP=B(NBGRW,J)
      B(NBGRW,J)=B(I,J)
   11 B(I,J)=TEMP
      ELIMINATE UNKNOWNS FROM FIRST COLUMN OF CURRENT SYSTEM
   12 DO 15 K=NN,NSIZ
      COMPUTE PIVOTAL MULTIPLIER
      PMULT=-A(K,I)*BG
      APPLY PMULT TO ALL COLUMNS OF THE CURRENT A-MATRIX ROW
      DO 13 J=NN,NSIZ
   13 A(K,J)=PMULT*A(I,J)+A(K,J)
      APPLY PMULT TO ALL COLUMNS OF MATRIX B
      DO 14 L=1,NBSIZ
   14 B(K,L)=PMULT*B(I,L)+B(K,L)
   15 CONTINUE
   16 CONTINUE
      DO BACK SUBSTITUTION
      WITH B-MATRIX COLUMN = NCOLB
   17 DO 22 NCOLB=1,NBSIZ
      DO FOR ROW = NROW
      DO 21 I=1,NSIZ
      NROW=NSIZ+1-I
```

```
      TEMP=0.0
      NUMBER OF PREVIOUSLY COMPUTED UNKNOWNS = NXS
      NXS=NSIZ-NROW
      ARE WE DOING THE BOTTOM ROW
      IF (NXS) 18,20,18
      NO
   18 DO 19 K=1,NXS
      KK=NSIZ+1-K
   19 TEMP=TEMP+B(KK,NCOLB)*A(NROW,KK)
   20 B(NROW,NCOLB)=(B(NROW,NCOLB)-TEMP)/A(NROW,NROW)
      HAVE WE FINISHED ALL ROWS FOR B-MATRIX COLUMN = NCOLB
   21 CONTINUE
      YES
      HAVE WE JUST FINISHED WITH B-MATRIX COLUMN NCOLB=NSIZ
   22 CONTINUE
      YES
      NOW FINISH COMPUTING THE DETERMINANT
      DO 23 I=1,NSIZ
   23 DET=DET*A(I,I)
      WE ARE ALL DONE NOW
      WHEW...
      RETURN
      END
      SUBROUTINE PRINT (X)
      COMMON DLTEMP
      COMMON TEMP,CA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CP,
     1DA,DB,H,CACB,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),RT(30),FNTOA,FN
     2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FM
     3X,FNOA,FNOB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DW
     4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DW
     5UA
      COMMON /SHEAR/ QA,QB,FQ
      REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2
      WRITE (9,4) X,HA,HB,ELL,R,TEMP
      WRITE (9,2) KLIP
      WRITE (9,5) WW,WWA
      WRITE (9,6) WB,WA
      WRITE (9,7) WU,WUA
      WRITE (9,8) TAU,WPJ
      WRITE (9,9) FNXA,FNXB,FMXA,FMXB
      WRITE (9,10) FNX,FMX
      WRITE (9,3) FNOA,FNOB

      WRITE (9,1) QA,QB,FQ
      RETURN


    1 FORMAT (1X,3HQA=E12.5,2X,3HQB=E12.5,2X,5HQBAR=E12.5/)
    2 FORMAT (1X,29H                            KLIP=I3)
    3 FORMAT (2X,5HFNOA=E12.5,2X,5HFNOB=E12.5)
    4 FORMAT (2X,2HX=F9.2,3HHA=F9.2,2X,3HHB=F9.2,2X,4HELL=F7.1,2X,2HR=F7
     1.1,2X,5HTEMP=E9.1/)
    5 FORMAT (2X,5HW(B)=E12.5,2X,5HW(A)=E12.5/)
    6 FORMAT (2X,8HBETA(B)=E12.5,2X,8HBETA(A)=E12.5/)
    7 FORMAT (2X,5HU(B)=E12.5,2X,5HU(A)=E12.5/)
    8 FORMAT (2X,4HTAU=E12.5,2X,5HP(J)=E12.5/)
    9 FORMAT (1X,6HNX(A)=E12.5,1X,5HNX(B)=E12.5,1X,6HMX(A)=E12.5,1X,6HMX
     1(B)=E12.5/)
   10 FORMAT (2X,5HNBAR=E12.5,2X,5HMBAR=E12.5)
      END
```

```
      SUBROUTINE RSLT (X)
      COMMON DLTEMP
      COMMON TEMP,EA,EAC,NUA,NUAC,EB,EBC,NUB,NUBC,HA,HB,R,G3A,G3B,CA,CB,
     1DA,DB,H,CACH,CAPCB,NUNU,GAMA,GAMB,K(40),A(55),G(7),BT(30),FNTOA,FN
     2TOB,BDE(600),LOAD1,AM(50),LOAD2,EN(6,7),FNXA,FNXB,FMXA,FMXB,FNX,FM
     3X,FNQA,FNQB,AC(3),KLIP,ELL,DELPH(20),V(6),AL(18),WW,DWW,DDWW,WB,DW
     4B,DDWB,DDDWB,WA,DWA,WU,DWU,DDWU,DDDWU,TAU,DTAU,WPJ,WWA,DWWA,WUA,DW
     5UA
      COMMON /SHEAR/ QA,QB,FQ
      REAL NUA,NUAC,NUB,NUBC,NUNU,K,LOAD1,LOAD2
      WWA=WW+BT(9)
      DWWA=DWW
      WUA=WU+(HA/2.)*WA+(HB/2.)*WB
      DWUA=DWU+(HA/2.)*DWA+(HB/2.)*DWB
      FNXA=CA*DWUA+(CA*NUA/R)*(WWA)-BT(5)/(1.-NUA)+BT(21)
      FNXB=CB*DWU+(CB*NUB/R)*(WW)-BT(6)/(1.-NUB)+BT(22)
      FNX=FNXA+FNXB
      FMXA=DA*DWA-BT(12)/(1.-NUA)+BT(18)
      FMXB=DB*DWB-BT(13)/(1.-NUB)+BT(19)
      FMX=FMXA+FMXB+(HA/2.+HB/2.)*FNXA
      QA=(HA/12.)*TAU+GAMA*(WA+DWW)
      QB=(HB/12.)*TAU+GAMB*(WB+DWW)
      FQ=QA+QB
      RETURN
      END
      SUBROUTINE BINGO (EN,V)
      DIMENSION DUMMY(6)
      DIMENSION EN(6,7), REN(6,6), C(6,1), V(6)
      DO 1 I=1,6
      DO 1 J=1,6
      REN(I,J)=EN(I,J)
      C(I,1)=EN(I,7)
    1 CONTINUE
      CALL LEQ (REN,C,6,1,6,6,DET)
      DO 2 I=1,6
      V(I)=C(I,1)
    2 CONTINUE
      DO 3 I=1,6
      DUMMY(I)=EN(I,1)*C(1,1)+EN(I,2)*C(2,1)+EN(I,3)*C(3,1)+EN(I,4)*C(4,
     11)+EN(I,5)*C(5,1)+EN(I,6)*C(6,1)-EN(I,7)
    3 CONTINUE
      WRITE (9,4)
      WRITE (9,5) (DUMMY(I),I=1,6)
      RETURN

    4 FORMAT (40H SOLUTION CHECK BY SUBSTITUTION           )
    5 FORMAT (1X,8HNBAR(0)=E12.5,1X,8HMBAR(0)=E12.5,1X,8HQBAR(0)=E12.5/,
     11X,9HNBAR(40)=E12.5,1X,9HMBAR(40)=E12.5,1X,9HQBAR(40)=E12.5//)
      END
```

APPENDIX B

Data

TABLE B.1

AVERAGE MATERIAL PROPERTIES FOR PG AND ATJ GRAPHITE

	PG	ATJ
E	3.1×10^{6} psi	2.26×10^{6} psi
ν	-0.21	+ 0.30
ν_c	+ 0.90	+ 0.25
R	varies with case	30 in.
L	40 in.	40 in.
α	$1.43 \times 10^{-6} \frac{in}{in - {}^{o}F}$	$4.25 \times 10^{-6} \frac{in}{in - {}^{o}F}$
α_c	$13.1 \times 10^{-6} \frac{in}{in - {}^{o}F}$	$4.25 \times 10^{-6} \frac{in}{in - {}^{o}F}$

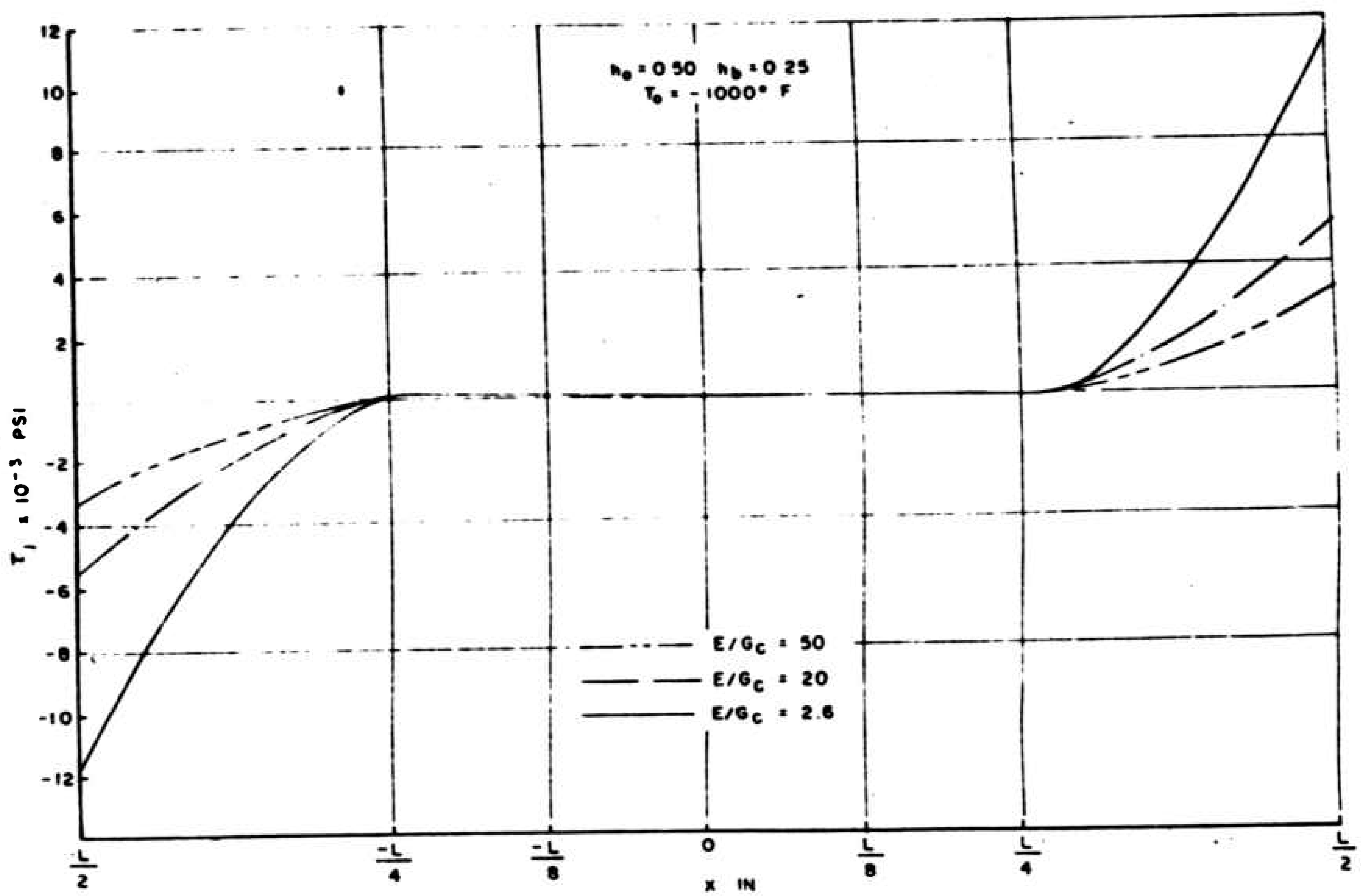

Figure B.1 Effect of E/G_c on Joint Shear Stress.

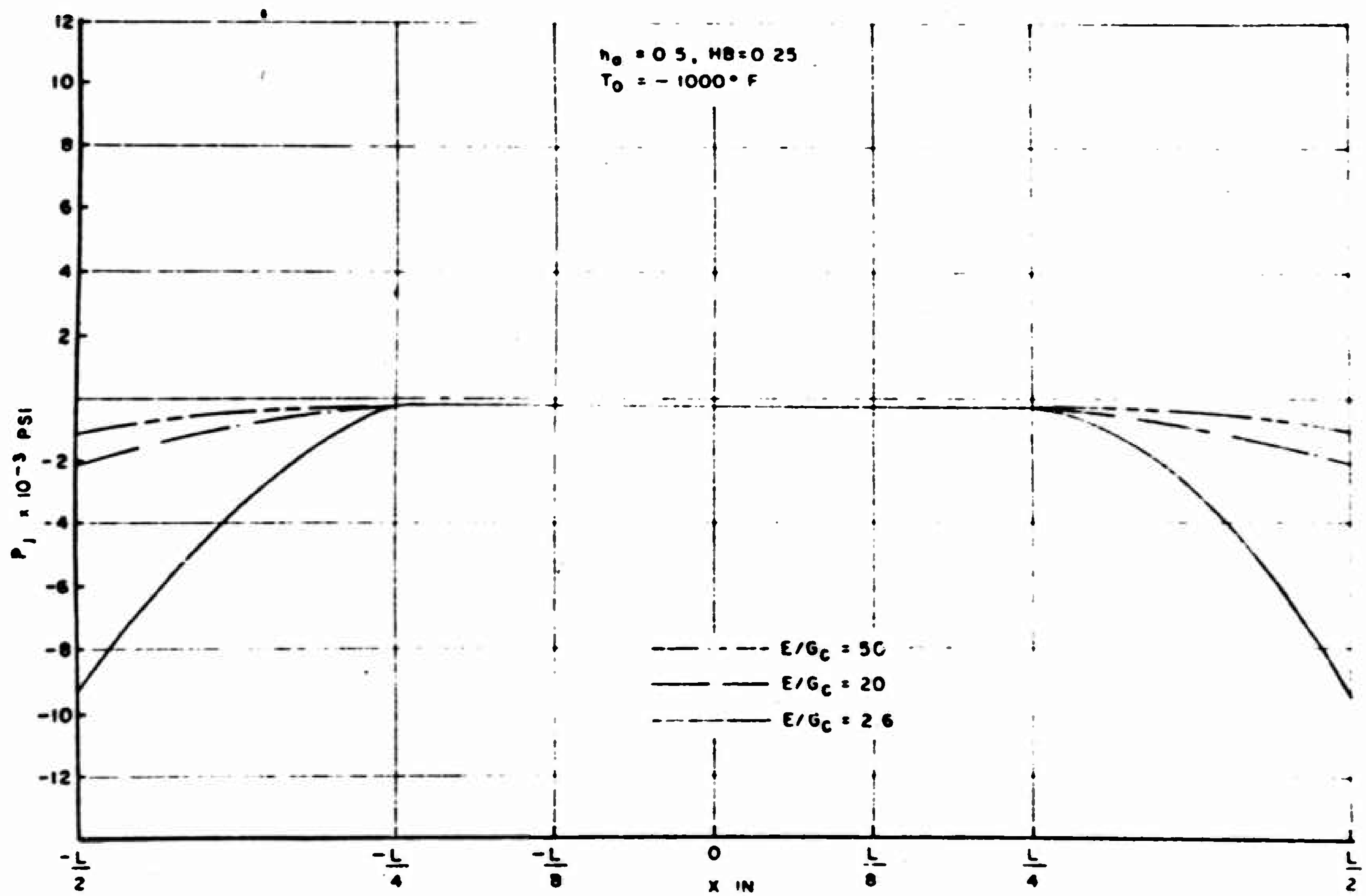

Figure B 2 Effect of E/G_c on Joint Normal Stress

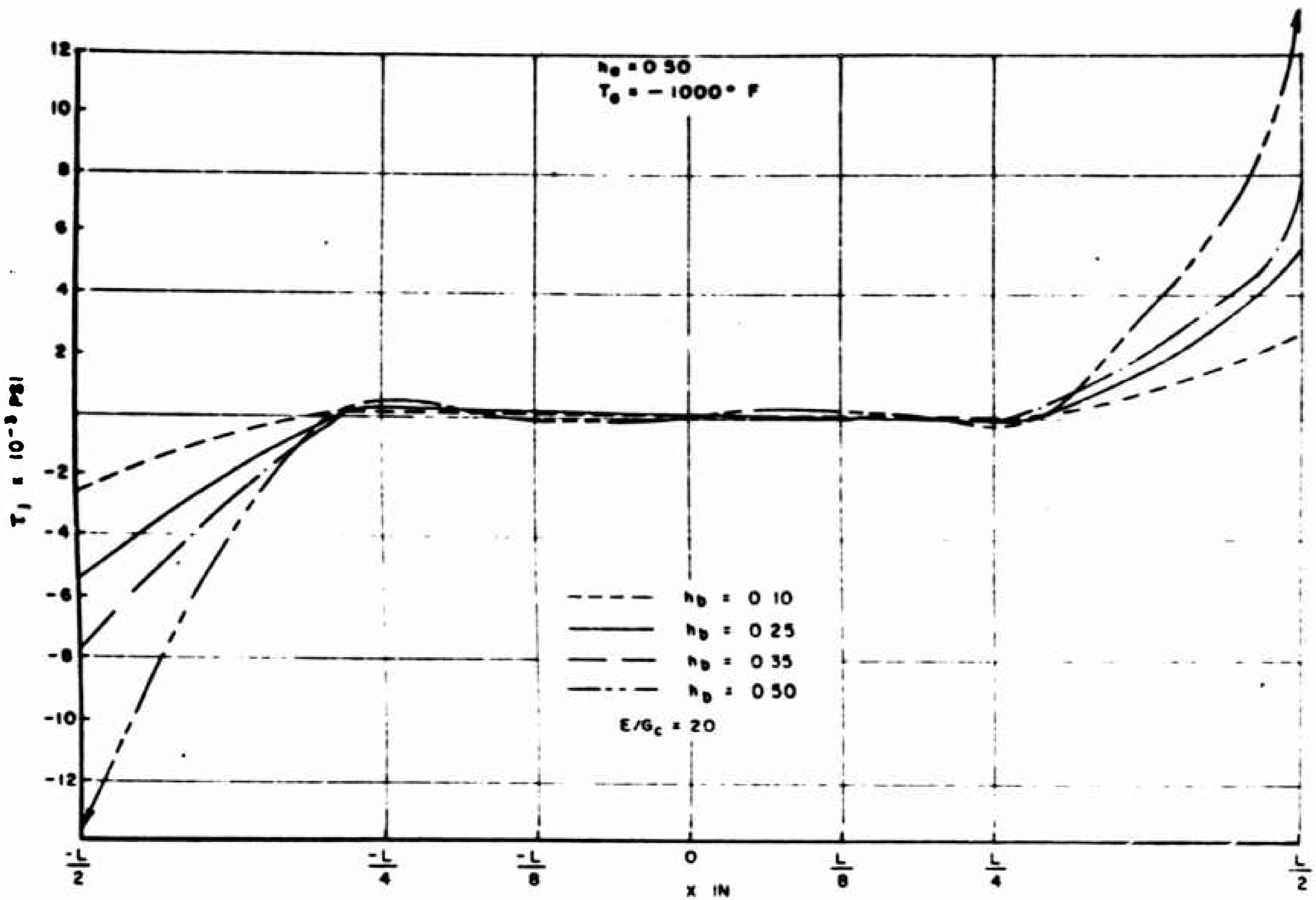

Figure B.3 Behavior of Joint Shear Stress With Varying Mandrel Thickness

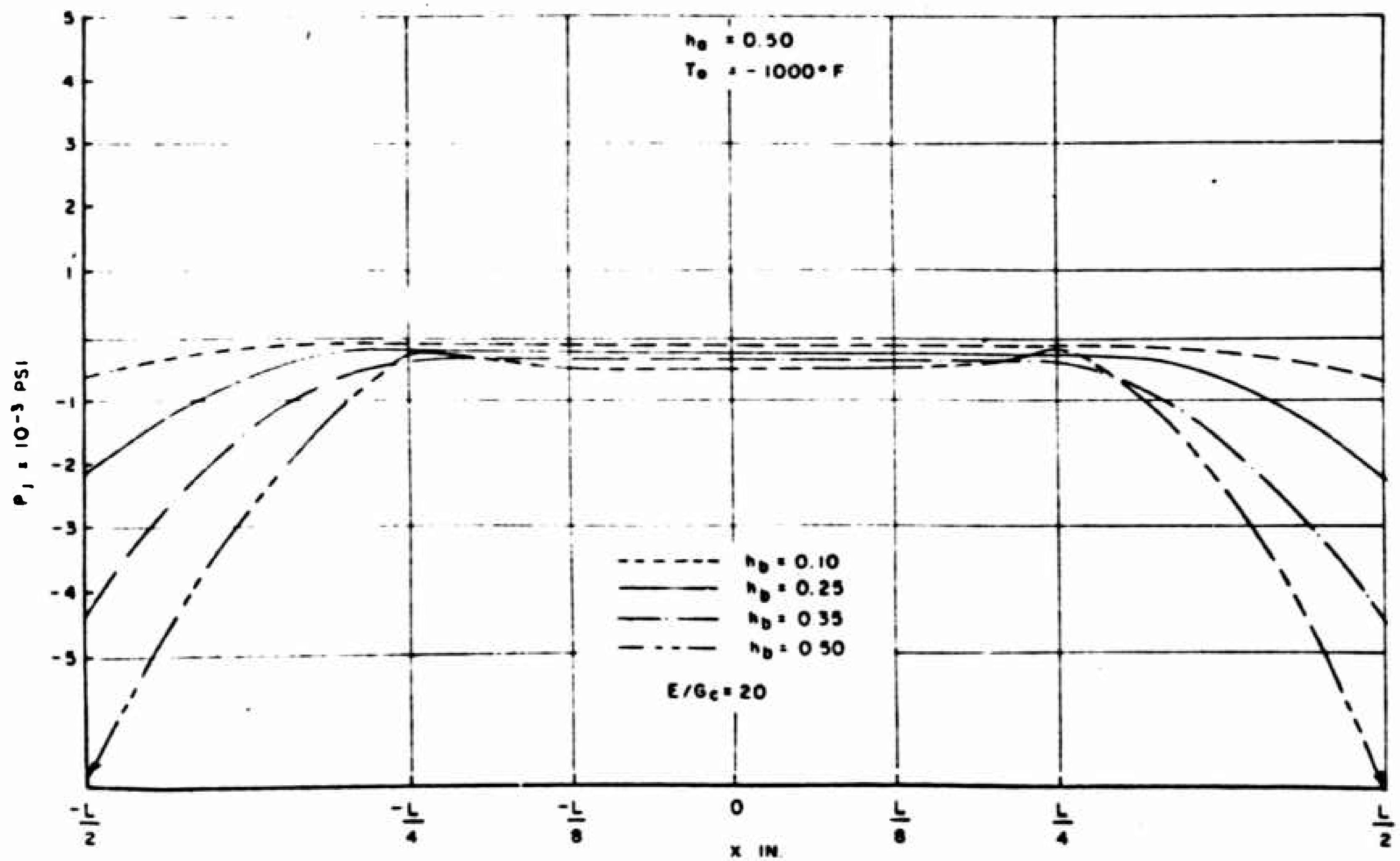

Figure B 4 Behavior of Joint Normal Stresses With Varying Mandrel Thickness

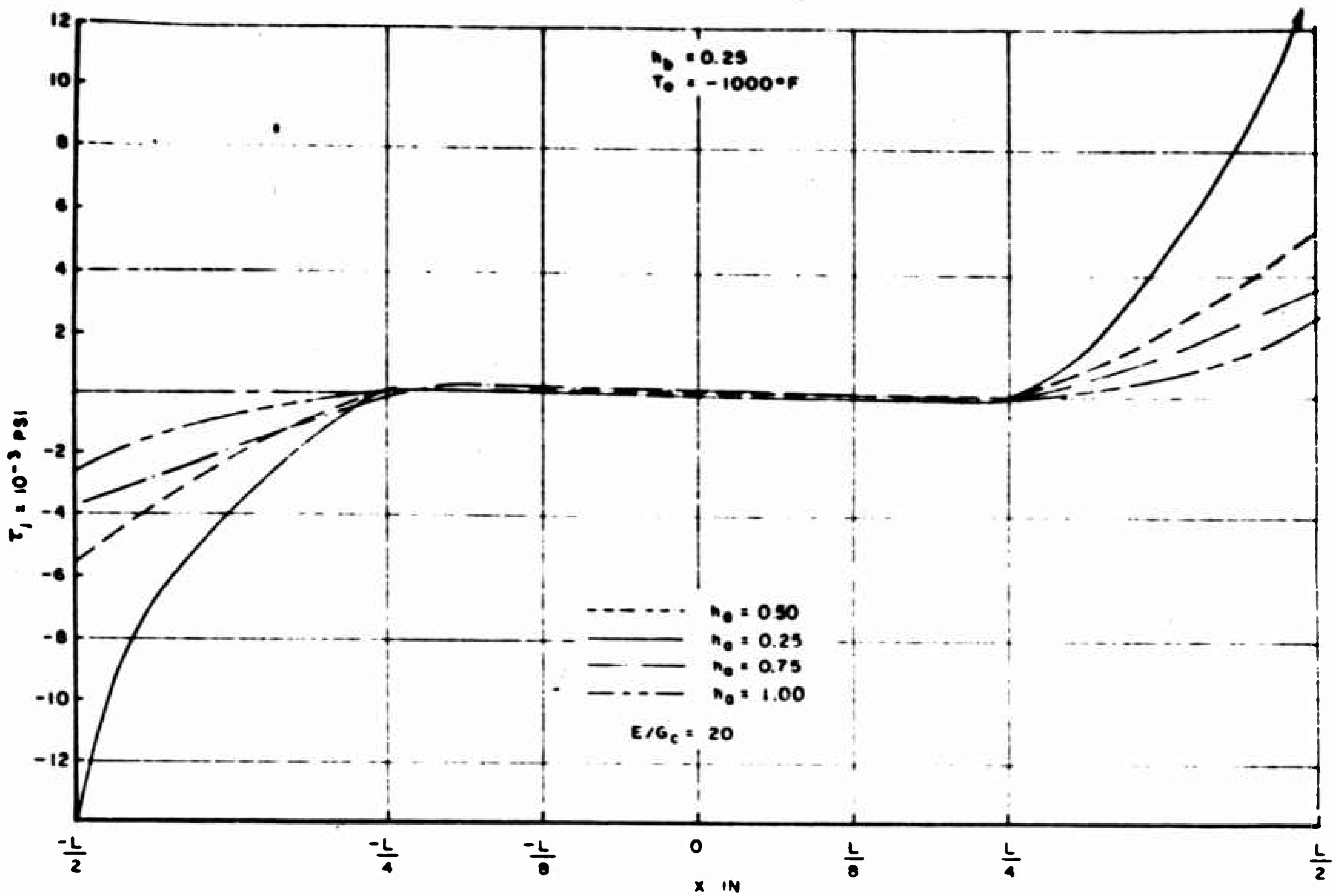

Figure B.5 Variation of Joint Shear Stresses With Increasing PG Deposition Thickness

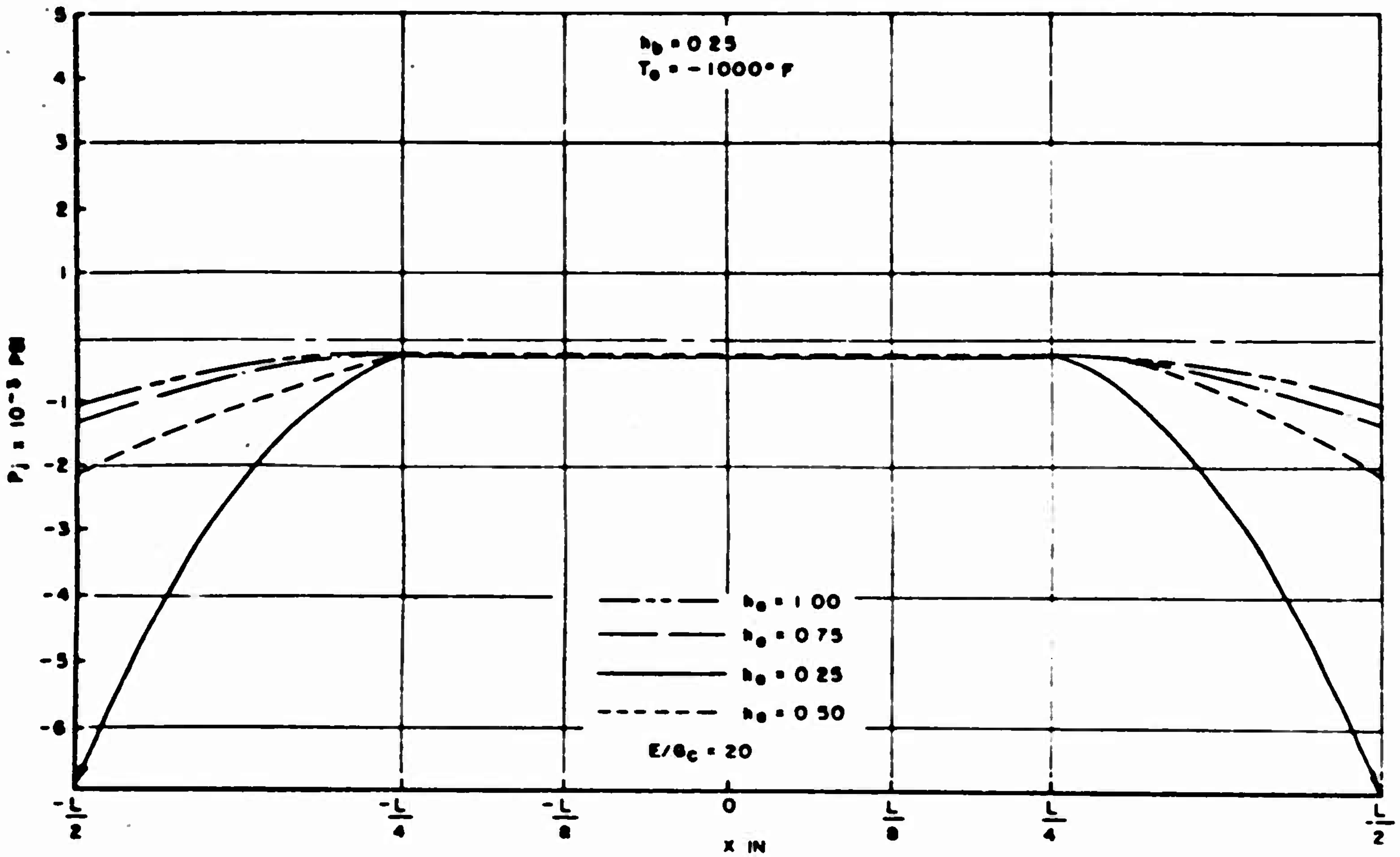

Figure B 6 Variation of Joint Normal Stresses With Increasing PG Deposition Thickness.

APPENDIX C

DEFINITION OF TERMS

$$Z_1 = (C_a + C_b)\ D\ \{(\alpha_{17} + \alpha_{27} + \pi_1 + \pi_2) - \frac{h}{2} D\ (\alpha_{47} + \pi_4)\}$$

$$+ \{\frac{h}{2} C_b D^2 + (\frac{C_a \nu_a + C_b \nu_b}{R_a\ R_b})\}\ (\alpha_{37} + \alpha_{47} + \pi_3 + \pi_4)$$

$$Z_2 = (C_a + C_b)\ \{\alpha_{67} + \pi_6 - \frac{5}{12} h_b\ (\alpha_{47} + \pi_4)\}$$

$$+ \frac{5}{12} h_b C_b\ (\alpha_{37} + \alpha_{47} + \pi_3 + \pi_4)$$

$$Z_3 = (C_a + C_b)\ \{\alpha_{57} + \pi_5 - \frac{5}{12} h_a\ (\alpha_{47} + \pi_4)\}$$

$$+ \frac{5}{12} h_a C_b\ (\alpha_{37} + \alpha_{47} + \pi_3 + \pi_4)$$

$$Z_4 = a_{31} Z_1 - (a_{11} D^2 + a_{15})\ Z_2$$

$$Z_5 = a_{23} D^2 Z_3 - (a_{31} D^2 + a_{32})\ Z_2 \qquad \text{(C.1)}$$

$$a_{11} = D_a\ (C_a + C_b) + \frac{C_a C_b h_a h}{4}$$

$$a_{12} = - \frac{C_a C_b h_a}{2}\ (\frac{\nu_a}{R_a} - \frac{\nu_b}{R_b})$$

$$a_{13} = D_b (C_a + C_b) + \frac{C_a C_b h_b h}{4}$$

$$a_{14} = \frac{h_b}{h_a} a_{12}$$

$$a_{15} = \frac{C_a C_b h}{2} \left(\frac{\nu_a}{R_a} - \frac{\nu_b}{R_b}\right)$$

$$a_{16} = \left(\frac{C_a \nu_a}{R_a} + \frac{C_b \nu_b}{R_b}\right)^2 - \left(\frac{C_a}{R_a} + \frac{C_b}{R_b}\right)^2$$

$$a_{21} = D_b (C_a + C_b) + \frac{5}{24} C_a C_b h_b^2$$

$$a_{22} = - (C_a + C_b) \frac{5}{6} G_c^b h_b$$

$$a_{23} = \frac{5}{24} C_a C_b h_a h_b$$

$$a_{24} = \frac{5}{12} C_a C_b h_b \left(\frac{\nu_a}{R_a} - \frac{\nu_b}{R_b}\right) + (C_a + C_b) \frac{5}{6} G_c^b h_b$$

$$a_{31} = D_a (C_a + C_b) + \frac{5}{24} C_a C_b h_a^2$$

$$a_{32} = - (C_a + C_b) \frac{5}{6} G_c^a h_a$$

$$a_{33} = a_{23}$$

$$a_{34} = \frac{5}{12} C_a C_b h_a \left(\frac{\nu_a}{R_a} - \frac{\nu_b}{R_b}\right) - \frac{5}{6} G_c^a h_a (C_a + C_b) \qquad (C.2)$$

$$b_{11} = a_{13}a_{23} - a_{11}a_{21}$$

$$b_{12} = a_{14}a_{23} - a_{11}a_{22} - a_{21}a_{12}$$

$$b_{13} = - a_{12}a_{22}$$

$$b_{14} = a_{15}a_{23} - a_{11}a_{24}$$

$$b_{15} = a_{16}a_{23} - a_{12}a_{24}$$

$$b_{21} = a_{23}a_{23} - a_{31}a_{21}$$

$$b_{22} = - (a_{31}a_{22} + a_{32}a_{21})$$

$$b_{23} = - a_{32}a_{22}$$

$$b_{24} = a_{23}a_{34} - a_{31}a_{24}$$

$$b_{25} = - a_{32}a_{24} \qquad (C.3\ )$$

$$g_1 = b_{21} b_{14} - b_{11} b_{24}$$

$$g_2 = b_{21} b_{15} + b_{22} b_{14} - b_{12} b_{24} - b_{11} b_{25}$$

$$g_3 = b_{22} b_{15} + b_{23} b_{14} - b_{12} b_{25} - b_{13} b_{24}$$

$$g_4 = b_{23} b_{15} - b_{13} b_{25} \qquad \text{(C.4)}$$

$$L_I(x) = (b_{21} D^4 + b_{22} D^2 + b_{23}) Z_4 - (b_{11} D^4 + b_{12} D^2 + b_{13}) Z_5$$

$$L_{II}(x) = Z_4$$

$$L_{III}(x) = Z_2$$

$$L_{IV}(x) = 1/(C_a + C_b)\ (\alpha_{37} + \alpha_{47} + \pi_3 + \pi_4)$$

$$L_V(x) = \alpha_{47} + \pi_4$$

$$L_{VI}(x) = \alpha_{27} + \pi_2 \qquad \text{(C.5)}$$

$$k_1 = C_a h_a / 2(C_a + C_b)$$

$$k_2 = C_a h_b / 2(C_a + C_b)$$

$$k_3 = 1/R\ (C_a \nu_a + C_b \nu_b)/(C_a + C_b)$$

$$k_4 = C_b$$

$$k_5 = k_8 = C_b \nu_b / R_b$$

$$k_6 - C_b h_b^2/12$$

$$k_7 = h_b/2$$

$$k_9 = C_b/R_b^2 \tag{C.6}$$

$$m = 1/3 \left\{ 3\left(\frac{g_3}{g_1}\right) - \left(\frac{g_2}{g_1}\right)^2 \right\}$$

$$n = 1/27 \left\{ 2\ \left(\frac{g_2}{g_1}\right)^3 - 9\ \left(\frac{g_2}{g_1}\right)\ \left(\frac{g_3}{g_1}\right) + 27\ \left(\frac{g_4}{g_1}\right) \right\} \tag{C.7}$$

$$A = \left\{ - n/2 + n^2/4 + m^3/27 \right\}^{1/3}$$

$$B = \left\{ - n/2 - n^2/4 + m^3/27 \right\}^{1/3} \tag{C.8}$$

$$\lambda_1 = A + B - 1/3\ (g_2/g_1)$$

$$\lambda_2 = +1/2\ (A + B) + 1/3\ (g_2/g_1) \tag{C.9}$$

$$\lambda_3 = \sqrt{\frac{3}{2}}\,(A - B)$$

$$\Gamma = (\lambda_2)^2 + (\lambda_3)^2$$

$$\tan\theta_1 = -\lambda_3 / \lambda_2$$

$$\tan\theta_2 = \lambda_3/\lambda_2 \qquad \text{(C.10)}$$

$$C_1 = (\lambda_1)^{1/2}$$

$$C_2 = \Gamma^{1/4} \cos(\theta_1/2) \qquad \text{(C.11)}$$

$$C_3 = \Gamma^{1/4} \sin(\theta_1/2)$$

$$q = \left| \frac{n^2}{4} + \frac{m^3}{27} \right|^{1/2} \qquad \text{(C.12)}$$

$$C_4 = (-n)^{1/3}$$

$$C_5 = -1/2 \left\{ C_4 + 3\,(2q)^{1/3} \right\} \qquad \text{(C.13)}$$

$$C_6 = -1/2 \left\{ C_4 - 3\,(2q)^{1/3} \right\}$$

APPENDIX D

Derivation of Integrated Shear Stress Resultant

The required shear stress-strain relation can be developed by weighted integration in order to obtain the factor 5/6 that is generally accepted for isotropic plates and shells. The method follows that in reference (15). For convenience, the procedure for cylindrical shells is reproduced here.

Expressions for normal stress distributions with z can be obtained by replacing the strains in the stress-strain relations (1) by the approximate forms (8) and neglecting $(z/R)_i$ in comparison to unity in the same terms. It is to be understood that the following hold for each lamina.

$$\sigma_x = \frac{E}{1-\nu^2}\left(u_o' + \frac{\nu w}{R} + z\beta'\right) - \frac{E\alpha T}{1-\nu} + \frac{\nu E\bar{w}}{R(1-\nu^2)} \tag{D.1}$$

$$\sigma_\theta = \frac{E}{1-\nu^2}\left(\nu u_o' + \frac{w}{R} + \nu z\beta'\right) - \frac{E\alpha T}{1-\nu} + \frac{E\bar{w}}{R(1-\nu^2)} \tag{D.2}$$

Expressions for the stress distributions in terms of stress resultants and couples can be obtained by using equations (11) in (D.1) and (D.2).

$$\sigma_x = \frac{N_x}{h} + \frac{N_{Tx}}{(1-\nu)h} - \frac{\nu E}{Rh(1-\nu^2)}\int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{w}dz + \frac{12}{h^3} z\left[M_x + \frac{M_{Tx}}{1-\nu} - \frac{E}{R(1-\nu^2)}\int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{w}zdz\right] - \frac{E\alpha T}{1-\nu} + \frac{\nu E\bar{w}}{R(1-\nu^2)} \tag{D.3}$$

$$\sigma_\theta = \frac{N_\theta}{h} + \frac{N_{T\theta}}{(1-\nu)h} - \frac{E}{Rh(1-\nu^2)}\int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{w}dz + \frac{12}{h^3}\, z \left[M_\theta + \frac{M_{T\theta}}{1-\nu} \right.$$

$$\left. - \frac{E}{Rh(1-\nu^2)}\int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{w}zdz \right] - \frac{E\alpha T}{1-\nu} + \frac{E}{R(1-\nu^2)}\bar{w} \tag{D.4}$$

The shear stress is related to the normal stress by the equilibrium equations (2). If the first of these is integrated with respect to z, the following is obtained.

$$\sigma_{xz} - \tau_{2i} = \frac{1}{R}\int_{-\frac{h}{2}}^{z} \frac{\partial}{\partial x}(R\sigma_x)\, dz \tag{D.5}$$

Using (D.3) in (D.5)

$$\sigma_{xz} - \tau_{2i} = -\frac{1}{Rh}\int_{-\frac{h}{2}}^{z} \left\{ RN_x' + \frac{12}{h^2}\, z\, RM_x' \right\} dz + \frac{1}{R}\int_{-\frac{h}{2}}^{z} \Omega\, dz \tag{D.6}$$

where

$$\Omega = \frac{R}{1-\nu}\frac{\partial}{\partial x}\left\{ \frac{N_{Tx}}{h} + \frac{12z}{h^3} M_{T_x} - E\alpha T - \frac{\nu E}{(1+\nu)R}\left[\frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{w}dz + \frac{12z}{h^3}\int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{w}zdz - \bar{w} \right] \right\}$$

Refering to the integrated equilibrium equations (10) and replacing the normal stress resultants and couples in (D.5) by their

equivalents in terms of the shear stress resultant.

$$\sigma_{xz} - \tau_{2i} = \frac{1}{h}\int_{-\frac{h}{2}}^{z}\left[-\frac{12z}{h^2}Q + \left(1 + \frac{6z}{h}\right)\tau_{1i} - \left(1 - \frac{6z}{h}\right)\tau_{2i}\right]dz + \frac{1}{R}\int_{-\frac{h}{2}}^{z}\Omega dz \qquad (D.8)$$

Performing the indicated integration

$$\sigma_{xz} - \tau_{2i} = \frac{3}{2}\left[1 - \left(\frac{2z}{h}\right)^2\right]\frac{Q}{h} - \frac{\tau_{1i}}{4}\left[1 - \frac{4z}{h} - 3\left(\frac{2z}{h}\right)^2\right] - \frac{\tau_{2i}}{4}\left[1 + \frac{4z}{h}\right.$$

$$\left. - 3\left(\frac{2z}{h}\right)^2\right] + \frac{1}{R}\int_{-\frac{h}{2}}^{z}\Omega dz \qquad (D.9)$$

From (D.6) and equations (9), it can readily be shown that

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\Omega dz = 0$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\Omega z dz = 0 \qquad (D.10)$$

The shear stress distribution (D.9) satisfies shear stress boundary conditions and the definition of the shear stress resultant. To prove the latter it is necessary to make use of (D.10) and

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-\frac{h}{2}}^{z} f(y)\, dy dz = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\frac{h}{2} - z)\, f(z)\, dz \qquad (D.11)$$

McDonough (15) points out that the effect of the last term in (D.9) is to modify the classical quadratic shear stress distribution but not the magnitude of the shear stress resultant. The modification is due to the nonlinearity of the normal stress distribution, primarily its distribution with x.

Proceeding with the weighted integration as in (15), the shear stress-strain relation of the set (1) is multiplied by the weighting function $\left[1 - (\frac{2z}{h})^2\right]$ and then integrated through the thickness of the shell to yield:

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \left[1 - (\frac{2z}{h})^2\right] \sigma_{xz} dz = 2G_c \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[1 - (\frac{2z}{h})^2\right] G_{xz} dz \qquad (D.12)$$

The integral on the left hand side is evaluated using (D.9) with the aid of (D.10) and (D.11). The integral on the right hand side is evaluated by using the shear strain given in equation (8). After integration, rearrangement and simplification, equation (11A) results:

$$Q = \frac{\overline{m}}{6} + \frac{5}{6} G_c h \left(\beta_i + w_i'\right) + \frac{5}{4} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[1 - \left(\frac{2z}{h}\right)^2 \right] G_c \overline{w}' dz + \frac{5}{4R} \int_{-\frac{h}{2}}^{\frac{h}{2}}$$

$$\left(\frac{2z}{h}\right)^2 \int_{-\frac{h}{2}}^{z} \Omega dy dz \qquad (D.13)$$

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) U.S. Army Aberdeen Research & Development Center Ballistic Research Laboratories Aberdeen Proving Ground, Maryland	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

Analysis of Transversely Isotropic Laminated Cylinders

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)

Jonas A. Zukas

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
November 1969	112	

8a. CONTRACT OR GRANT NO. b. PROJECT NO. 1B062104A088 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) Report No. 1457 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	U.S.Army Materiel Command Washington,D.C.

13. ABSTRACT

A theory for the analysis of stresses in laminated circular cylindrical shells subjected to arbitrary axisymmetric mechanical and thermal loadings has been developed. This theory, specifically for use with pyrolytic graphite type materials, differs from the classical thin shell theory in that it includes the effects of transverse shear deformation and transverse isotropy, as well as thermal expansion through the shell thickness.

Solutions in several forms are developed for the governing equations. The form taken by the solution function is governed by geometric considerations. A range in which the various solution forms occur was determined numerically.

As a sample problem, the slow cooling of pyrolytic graphite deposited onto a commercial graphite mandrel was considered. Investigation of normal and shear stress behavior at the pyrolytic graphite - mandrel interface showed that these stresses decrease in magnitude with increasing E/G_c ratio and increasing deposit to mandrel thickness (h_a/h_b)ratio. This implies that a thin mandrel and a material weak in shear are desirable to minimize the possibilities of flaking and delamination of the pyrolytic graphite.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED
Security Classification

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Thermoelasticity						
Laminated shells						
Pyrolytic graphite						
Transverse Isotropy						
Thermal expansion						
Shells						
Cylinders						

Security Classification